

Parametric structural schematic diagram of electroelastic actuator in nanotechnology.

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Abstract

In this work the parametric structural schematic diagram of the electroelastic actuator or the piezoactuator is determined in contrast the electrical equivalent circuit types Cady or Mason for the calculation of the piezoelectric transmitter and receiver, the vibration piezomotor with the mechanical parameters in form the velocity and the pressure. The method of mathematical physics is used. The parametric structural schematic diagram of electroelastic actuator is obtained with the mechanical parameters the displacement and the force. The transfer functions of the electroelastic actuator are determined. The generalized parametric structural schematic diagram, the generalized matrix equation for the electroelastic actuator nano displacement are obtained. The deformations of the electroelastic actuator for the nanotechnology are described by the matrix equation.

Keywords: Electroelastic actuator, Parametric structural schematic diagram, Transfer function, Piezoactuator.

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Introduction

The parametric structural schematic diagram of the electroelastic actuator on the piezoelectric, piezomagnetic, electrostriction effects, for example, the piezoactuator is determined in contrast electrical equivalent circuit types Cady or Mason for the calculation of the piezotransmitter and piezoreceiver, the vibration piezomotor with the mechanical parameters in form the velocity and the pressure [1-8]. The parametric structural schematic diagram of the actuator is obtained with the mechanical parameters the displacement and the force. The electroelastic actuator for nanotechnology is used in the scanning tunneling microscopes, the scanning force microscopes, the atomic force microscopes [8-10].

Method

The method of mathematical physics is applied for the solution of the wave equation of the electroelastic actuator for the nanotechnology with using the Laplace transform for the construction the parametric structural schematic diagram of electroelastic actuator [11-16].

As the result of the joint solution of the wave equation of the actuator with the Laplace transform, the equation of the electroelasticity, the boundary conditions on the two loaded working surfaces of the actuator, we obtain the corresponding structural-parametric model and the parametric structural schematic diagram of electroelastic actuator.

Results and Discussion

The parametric structural schematic diagram and the matrix transfer functions of the electroelastic actuator for the nanotechnology are obtained from the structural-parametric model of the electroelastic actuator with the mechanical

parameters the displacement and the force. The structural diagrams of the voltage-controlled or current-controlled piezoactuator are determined from the generalized structural-parametric model of the electroelastic actuator [17,18].

Generalized parametric structural schematic diagram

Let us consider the generalized equation of the electroelasticity [8,11,18] in the form:

$$S_i = d_{mi} \Psi_m(t) + s_{ij}^{\Psi} T_j(x, t) \quad (1)$$

where $S_i = \partial \xi(x, t) / \partial x$ is the relative displacement along axis i of the cross section of the piezoactuator or the piezoplate, $\Psi_m = \{E_m, D_m\}$ is the control parameter E for the voltage control, D for the current control along axis m , T_j is the mechanical stress along axis j , d_{mi} is the coefficient of electroelasticity, for example, piezomodule, s_{ij}^{Ψ} is the elastic compliance for control parameter $\Psi = \text{const}$, indexes $i, j = 1, 2, \dots, 6$; $m = 1, 2, 3$.

The main size is determined us the working length $l = \{\delta, h, b\}$ for the electroelastic actuator or the piezoactuator in form the thickness, the height and the width for the longitudinal, transverse and shift the piezo effect.

For the construction the parametric structural schematic diagram of electroelastic actuator in nanotechnology is used the wave equation [8,11,18] for the wave propagation in a long line with damping but without distortions. With using Laplace transform is obtained the linear ordinary second-order differential equation with the parameter p . Correspondingly the original problem for the partial differential equation of hyperbolic type using the Laplace transform is reduced to the simpler problem [8,11,12] for the linear ordinary differential equation

$$\frac{d^2 \Xi(x, p)}{dx^2} - \gamma^2 \Xi(x, p) = 0 \quad (2)$$

with its solution

$$\Xi(x, p) = Ce^{-\gamma x} + Be^{\gamma x} \quad (3)$$

where $\Xi(x, p)$ is the Laplace transform of the displacement of section of the actuator, $\gamma = p/c^\Psi + \alpha$ is the propagation coefficient, c^Ψ is the sound speed for $\Psi = const$, α is the damping coefficient, C and B are constants.

The generalized structural-parametric model and the generalized parametric structural schematic diagram of the electroelastic actuator for nanotechnology on Figure 1 are determined, using equations (1) and (2), the boundary conditions on loaded faces and the strains along the axes, in the following form

$$\begin{aligned} \Xi_1(p) &= [1/(M_1 p^2)] \times \\ &\times \left\{ -F_1(p) + (1/\chi_{ij}^\Psi) [v_{mi} \Psi_m(p) - [\gamma/\text{sh}(l\gamma)] [\text{ch}(l\gamma)\Xi_1(p) - \Xi_2(p)]] \right\} \\ \Xi_2(p) &= [1/(M_2 p^2)] \times \\ &\times \left\{ -F_2(p) + (1/\chi_{ij}^\Psi) [v_{mi} \Psi_m(p) - [\gamma/\text{sh}(l\gamma)] [\text{ch}(l\gamma)\Xi_2(p) - \Xi_1(p)]] \right\} \end{aligned} \quad (4)$$

$$\text{where } \chi_{ij}^\Psi = \frac{s_{ij}^\Psi}{S_0}, \quad v_{mi} = \begin{cases} d_{33}, d_{31}, d_{15} \\ g_{33}, g_{31}, g_{15} \end{cases}, \quad \Psi_m = \begin{cases} E_3, E_3, E_1 \\ D_3, D_3, D_1 \end{cases}$$

$$s_{ij}^\Psi = \begin{cases} s_{33}^E, s_{11}^E, s_{55}^E \\ s_{33}^D, s_{11}^D, s_{55}^D \end{cases}, \quad l = \{ \delta, h, b \}, \quad c^\Psi = \{ c^E, c^D \}, \quad \gamma^\Psi = \{ \gamma^E, \gamma^D \}$$

v_{mi} is the coefficient of the electroelasticity, for example, d_{mi} is the piezomodule for the voltage-controlled piezoactuator, g_{mi} is the piezomodule for the current-controlled piezoactuator, S_0 is the cross section area and M_1, M_2 are the displaced mass on the faces of the electroelastic actuator, $F_1(p), F_2(p)$ and $F_l(p)$ are the Laplace transform of the displacements and

the forces on the faces of the electroelastic actuator, Ψ_m is the control parameter of the electroelastic actuator.

The matrix state equations [2,11,14] for the piezoelectric effect have the form:

$$(\mathbf{D}) = (\mathbf{d})(\mathbf{T}) + (\epsilon^T)(\mathbf{E}) \quad (5)$$

$$(\mathbf{S}) = (\mathbf{s}^E)(\mathbf{T}) + (\mathbf{d}^T)(\mathbf{E}) \quad (6)$$

where the first equation describes the direct piezoeffect, the second equation presents the inverse piezoeffect, (\mathbf{D}) is the column-matrix of the electric induction along the coordinate axes, (\mathbf{S}) is the column-matrix of the relative deformations, (\mathbf{T}) is the column-matrix of the mechanical stresses, (\mathbf{E}) is the column-matrix of the electric field strength along the coordinate axes, (\mathbf{d}^T) is the transposed matrix of the piezoelectric modules, (\mathbf{s}^E) is the elastic compliance matrix, (ϵ^T) is the matrix of dielectric constants.

The deformation of the piezoactuator corresponds to its stressed state. If the mechanical stress T are created in the piezoelement, the deformations S are formed in it [8,11].

There are the six stress components $T_1, T_2, T_3, T_4, T_5, T_6$. The components T_3-T_3 are defined to extension-compression stresses, the components T_4-T_6 are related to shear stresses.

Let us consider the transverse piezoelectric effect in the piezoactuator shown in Figure 2. The equation of the inverse transverse piezoeffect [8,11]

$$S_1 = d_{31} E_3(t) + s_{11}^E T_1(x, t) \quad (7)$$

where $S_1 = \partial \xi(x, t) / \partial x$ is the relative displacement of the cross section of the piezoactuator along axis 1, d_{31} is the piezoelectric module for the transverse piezoeffect, s_{11}^E is the elastic compliance for $E = const$ along axis 1, T_1 is the stress along axis 1.

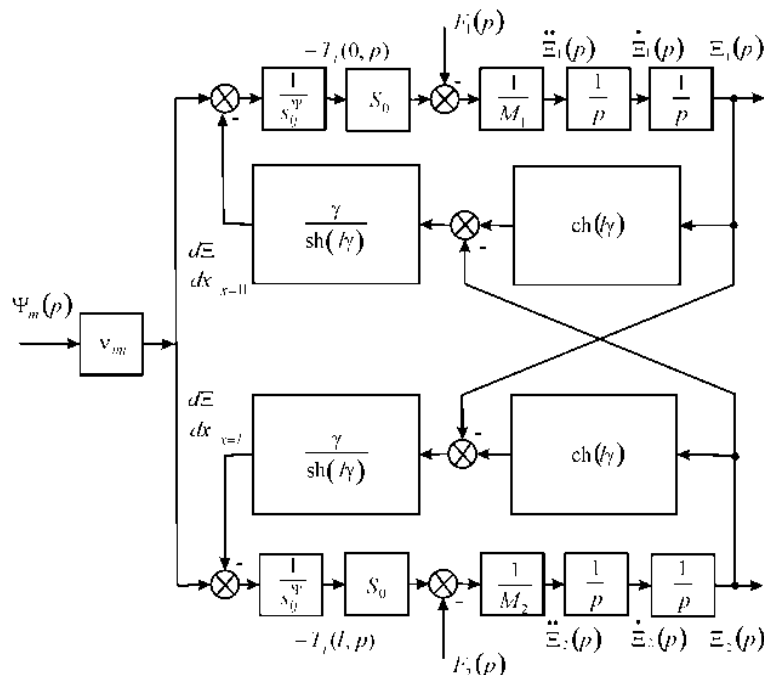


Figure 1. Generalized parametric structural schematic diagram of electroelastic actuator in nanotechnology.

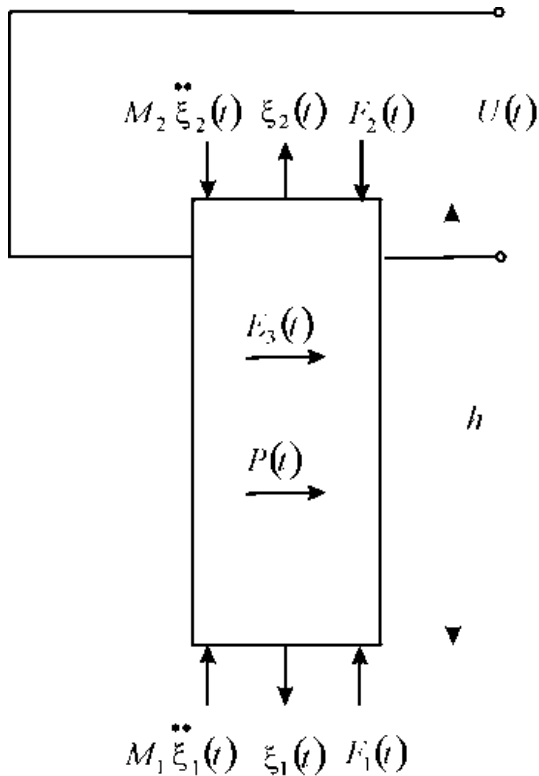


Figure 2. Piezoactuator for the transverse piezoelectric effect.

The solution of the linear ordinary second-order differential equation with the parameter p (2) can be written as (3) and subject to the conditions

$$\begin{aligned} \Xi(0, p) &= \Xi_1(p) \text{ for } x=0 \\ \Xi(l, p) &= \Xi_2(p) \text{ for } x=h \end{aligned} \quad (8)$$

Therefore the constants C and B for the solution we obtain in the following form:

$$C = (\Xi_1 e^{h\gamma} - \Xi_2) / [2\text{sh}(h\gamma)], \quad B = (\Xi_2 - \Xi_1 e^{-h\gamma}) / [2\text{sh}(h\gamma)] \quad (9)$$

Then, the solution (3) can be written as:

$$\Xi(x, p) = \{\Xi_1(p)\text{sh}[(h-x)\gamma] + \Xi_2(p)\text{sh}(x\gamma)\} / \text{sh}(h\gamma) \quad (10)$$

The equations of forces acting on the faces of the piezoactuator has the form:

$$\begin{aligned} T_1(0, p)S_0 &= F_1(p) + M_1 p^2 \Xi_1(p) \text{ for } x=0 \\ T_1(h, p)S_0 &= -F_2(p) - M_2 p^2 \Xi_2(p) \text{ for } x=h \end{aligned} \quad (11)$$

where $T_1(0, p)$ and $T_1(h, p)$ are determined from the equation of the inverse transverse piezoeffect.

Therefore we obtain the system of the equations for the mechanical stresses at the faces of the piezoactuator for the transverse piezoeffect in the form:

$$\begin{aligned} T_1(0, p) &= \frac{1}{s_{11}^E} \frac{d\Xi(x, p)}{dx} \Big|_{x=0} - \frac{d_{31}}{s_{11}^E} E_3(p) \\ T_1(h, p) &= \frac{1}{s_{11}^E} \frac{d\Xi(x, p)}{dx} \Big|_{x=h} - \frac{d_{31}}{s_{11}^E} E_3(p) \end{aligned} \quad (12)$$

The set of equations (12) for mechanical stresses in piezoactuator yields the following set of equations describing the structural parametric model and parametric structural

schematic diagram of the voltage-controlled piezoactuator for the transverse piezoelectric effect on Figure 3.

$$\begin{aligned} \Xi_1(p) &= [1/(M_1 p^2)] \times \\ &\times \{-F_1(p) + (1/\chi_{11}^E) [d_{31} E_3(p) - [\gamma/\text{sh}(h\gamma)] [\text{ch}(h\gamma)\Xi_1(p) - \Xi_2(p)]]\} \end{aligned} \quad (13)$$

$$\begin{aligned} \Xi_2(p) &= [1/(M_2 p^2)] \times \\ &\times \{-F_2(p) + (1/\chi_{11}^E) [d_{31} E_3(p) - [\gamma/\text{sh}(h\gamma)] [\text{ch}(h\gamma)\Xi_2(p) - \Xi_1(p)]]\} \end{aligned}$$

$$\text{where } \chi_{11}^E = s_{11}^E / S_0, \quad l = h$$

The parametric structural schematic diagrams of the voltage-controlled or current-controlled piezoactuator for the transverse, longitudinal, shift piezoelectric effects are determined from the generalized structural-parametric model of the electroelastic actuator.

Matrix equation of displacements

From generalized structural-parametric model of the electroelastic actuator, taking into account the generalized equation of the electroelasticity, wave equation and the equation of the forces on its faces, we obtain the transfer functions of the electroelastic actuator.

Correspondingly the Laplace transforms of displacements for two faces of the electroelastic actuator are dependent from the Laplace transforms of the general parameter of control and forces on two faces. Matrix equation of the Laplace transforms of the displacements with the matrix transfer functions of the electroelastic actuator is obtained [8,14,18] in the form

$$\begin{pmatrix} \Xi_1(p) \\ \Xi_2(p) \end{pmatrix} = \begin{pmatrix} W_{11}(p) & W_{12}(p) & W_{13}(p) \\ W_{21}(p) & W_{22}(p) & W_{23}(p) \end{pmatrix} \begin{pmatrix} \Psi_m(p) \\ F_1(p) \\ F_2(p) \end{pmatrix} \quad (14)$$

Let us consider the displacements the faces of the voltage-controlled the piezoactuator for the transverse piezoeffect $\Psi_m = E_3$ with the output parameter displacement.

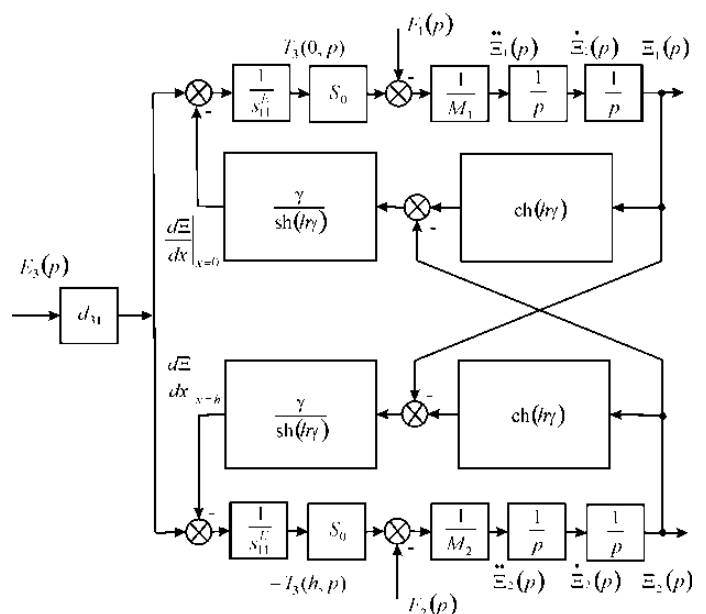


Figure 3. Parametric structural schematic diagram of voltage-controlled piezoactuator for transverse piezoelectric effect.

In matrix equation (14) the transfer functions of the voltage-controlled the piezoactuator for the transverse piezoeffect can be written in the following form:

$$W_{11}(p) = \Xi_1(p)/E_3(p) = d_{31} [M_2 \chi_{11}^E p^2 + \gamma \text{th}(h\gamma/2)] / A_{11}$$

$$\chi_{11}^E = s_{11}^E / S_0$$

$$A_{11} = M_1 M_2 (\chi_{11}^E)^2 p^4 + \{ (M_1 + M_2) \chi_{11}^E / [c^E \text{th}(h\gamma)] \} p^3 +$$

$$+ [(M_1 + M_2) \chi_{11}^E \alpha / \text{th}(h\gamma) + 1 / (c^E)^2] p^2 + 2\alpha p / c^E + \alpha^2$$

$$W_{21}(p) = \Xi_2(p)/E_3(p) = d_{31} [M_1 \chi_{11}^E p^2 + \gamma \text{th}(h\gamma/2)] / A_{11}$$

$$W_{12}(p) = \Xi_1(p)/F_1(p) = -\chi_{11}^E [M_2 \chi_{11}^E p^2 + \gamma / \text{th}(h\gamma)] / A_{11}$$

$$W_{13}(p) = \Xi_1(p)/F_2(p) =$$

$$= W_{22}(p) = \Xi_2(p)/F_1(p) = [\chi_{11}^E \gamma / \text{sh}(h\gamma)] / A_{11}$$

$$W_{23}(p) = \Xi_2(p)/F_2(p) = -\chi_{11}^E [M_1 \chi_{11}^E p^2 + \gamma / \text{th}(h\gamma)] / A_{11}$$

The static displacements of the faces for the voltage-controlled the piezoactuator for the transverse piezoeffect are obtained from (14) at $m \ll M_1$ and $m \ll M_2$ in the form

$$\xi_1(\infty) = \lim_{\substack{p \rightarrow 0 \\ \alpha \rightarrow 0}} \frac{p W_{11}(p) U_0}{\delta p} = \frac{d_{31} h U_0 M_2}{\delta (M_1 + M_2)} \quad (15)$$

$$\xi_2(\infty) = \lim_{\substack{p \rightarrow 0 \\ \alpha \rightarrow 0}} \frac{p W_{21}(p) U_0}{\delta p} = \frac{d_{31} h U_0 M_1}{\delta (M_1 + M_2)} \quad (16)$$

For the voltage-controlled the piezoactuator from PZT under the transverse piezoeffect at $m \ll M_1$ and $m \ll M_2$, $d_{31} = 2.5 \cdot 10^{-10}$ m/V, $h/\delta = 20$, $U = 50$ V, $M_1 = 2$ kg and $M_2 = 8$ kg the static displacements of the faces the piezoactuator are determined $\xi_1(\infty) = 200$ nm, $\xi_2(\infty) = 50$ nm, $\xi_1(\infty) + \xi_2(\infty) = 250$ nm.

The transfer function of the voltage-controlled transverse piezoactuator is obtained from (14) for elastic-inertial load at $M_1 \rightarrow \infty$, $m \ll M_2$ and the approximation the hyperbolic cotangent by two terms of the power series in the form:

$$W(p) = \frac{\Xi_2(p)}{U(p)} = \frac{d_{31} h / \delta}{(1 + C_e / C_{11}^E) (T_i^2 p^2 + 2T_i \xi_i p + 1)} \quad (17)$$

$$T_i = \sqrt{M_2 / (C_e + C_{11}^E)}, \quad \xi_i = \alpha h^2 C_{11}^E / (3c^E \sqrt{M_2 (C_e + C_{11}^E)})$$

where $U(p)$ is the Laplace transform of the voltage, T_i is the time constant and ξ_i is the damping coefficient of the piezoactuator.

Therefore the expression for the transient response of the voltage-controlled transverse piezoactuator is determined in the following form:

$$\xi(t) = \xi_m \left[1 - \frac{e^{-\frac{\xi_i t}{T_i}}}{\sqrt{1 - \xi_i^2}} \sin(\omega_t t + \phi_t) \right] \quad (18)$$

$$\xi_m = \frac{d_{31} (h/\delta) U_m}{1 + C_e / C_{11}^E}, \quad \omega_t = \frac{\sqrt{1 - \xi_i^2}}{T_i}, \quad \phi_t = \arctg \left(\frac{\sqrt{1 - \xi_i^2}}{\xi_i} \right)$$

where ξ_m is the steady-state value of displacement for the voltage-controlled piezoactuator, U_m is the amplitude of the voltage in the steady-state. For $M_1 \rightarrow \infty$, $m \ll M_2$, $U_m = 150$ V, $d_{31} = 2.5 \cdot 10^{-10}$ m/V, $h/\delta = 20$, $M_2 = 1$ kg, $C_{11}^E = 2 \cdot 10^7$ N/m, $C_e = 0.5 \cdot 10^7$ H/m we obtain values the steady-state value of displacement and the time constant of the actuator $\xi_m = 600$ nm, $T_i = 0.2 \cdot 10^{-3}$ c.

The matrix transfer functions of the actuator are determined for control systems with the electroelastic actuator in the nanotechnology.

In this work the generalized parametric structural schematic diagram and generalized structural-parametric model of the electroelastic actuator are obtained. From generalized structural-parametric model of the electroelastic actuator after algebraic transformations the transfer functions of the electroelastic actuator are determined.

The parametric structural schematic diagrams, the structural-parametric models of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are determined from the generalized structural-parametric model of the electroelastic actuator for nanotechnology.

Conclusion

For nanotechnology the generalized parametric structural schematic diagram and the generalized structural-parametric model of the electroelastic actuator are constructed with the mechanical parameters the displacement and the force.

The parametric structural schematic diagrams of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are determined.

The matrix transfer functions of the electroelastic actuator are determined for control systems in the nanotechnology.

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