Applications of the HWM and the MOL for solving a non-linear tsunami model of coupled partial differential equations.

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Abstract

This paper presents two strategies for getting the answers for a Non-Linear Tsunami Model of Coupled Partial Differential Equations. The first is the Haar Wavelet technique (HWM). The second technique is the method of lines (MOL). The numerical consequences of the MOL are contrasted and the after effects of the HWM. To demonstrate the unwavering quality of the considered techniques we have contrasted the gotten arrangements and the correct ones. The outcomes uncover that the two techniques are powerful and helpful for settling such sorts of halfway differential conditions, however the strategy for lines gives less precise outcomes over the other strategy.

Keywords: Method of Lines (MOL), Haar Wavelet technique (HWM), Numerical solutions.

Introduction

Wave is a Japanese word with the English translation, "harbor wave". Addressed by two characters, the best character, "tsu" suggests harbor, while the base character, "nami" implies "wave." previously, tsunamis were currently insinuated as "tidal waves" by the general populace, and as "seismic sea waves" by standard scientists. The articulation "tidal wave" is a misnomer; even though a tsunami's impact upon a coastline is poor upon the tidal level at the time a deluge strikes, tsunamis are detached to the tides. Tides result from the imbalanced, extra natural, gravitational effects of the moon, sun, and planets. The articulation "seismic sea wave" is moreover tricky. "Seismic" recommends a tremor related age framework, anyway a downpour can in like manner be caused by a no seismic event, for instance, a torrential slide or meteorite influence.

Methodology

Methods for numerical simulation of tsunami wave propagation in deep and shallow seas are well developed and furthermore, utilized by numerous researchers [1-8]. We simply provide the mathematical framework of Haar wavelet method and MOL for a non-linear tsunami model of coupled partial differential equations (PDEs).

Solution of Tsunami equation

The correct arrangement has been stood out from the *MOL* arrangements just when the variable ocean profundity *H(x)* is equivalent to the ocean profundity in the vast sea *(d)*. At that point the impact of increment the slant, the profundity of the water and the wave stature on the speed of tidal wave and the rise have been considered. At the point when a torrent approaches a coastline, the wave starts to back off and increment in tallness relying upon the geology of the ocean depths.

Solving Tsunami equation using the MOL: Consider the instance of the run-up of 2D long wave's occurrence upon a uniform inclining shoreline associated with a vast sea of uniform profundity (Figure 1). The traditional nonlinear shallow-water conditions are [6]:

$$u_t + uu_x + g\eta_x = 0$$

$$\eta_t + (u(H+\eta))_x = 0 \tag{1}$$

where as shown in Figure 1.

$u(x, t)$: is tsunami velocity; $\eta(x, t)$: is surface elevation (wave amplitude)

$H(x)$ is the variable sea depth; g: is the acceleration due to gravity g=9.8m/s^2

x: is the horizontal distance;

t: is the time;

The initial conditions can be written as

$$\eta(x,0) = h * \sec h^2 \sqrt{\frac{3h}{4d^3}} x$$

$$u(x,0) = \eta\sqrt{\frac{g}{d}} \tag{2}$$

Where, h: is an initial wave height; d: is the sea depth in the open ocean.

The exact solutions occur only when $H(x) = d$ they are:

$$\eta(x,t) = h * \sec h^2 \sqrt{\frac{3h}{4d^3}}(x - \sqrt{\frac{g}{d}}(1+0.5(\frac{h}{d}))t); \; u(x,t) = \eta\sqrt{\frac{g}{d}} \tag{3}$$

The system (1) has no exact solution when $H(x)$ is variable. So, we will solve this system for three different function of $H(x)$ in the form $H(x) = ax + 300$ which $H(x) = 0.5x + 300$, $H(x) = x + 300$ and $H(x) = 2x + 300$ to show the effect of the slop (a) on the run-up heights and the velocity of tsunami wave.

Tsunamis stages are: the first is formation, the second is mid-ocean propagation, and last is breaking and run-up on the beach.

Let x, t be the independent variables and u_1, u_2, u_3, … … . u_n, η_1, η_2, η_3, …. η_m be the dependent variables associated with a system of PDEs. The general form of a system of PDEs having 1st order time derivatives and 1st order space derivatives may be written as follows:

$$\frac{du_i(t)}{dt} = f(u_i, \eta_i) \; \forall \, i = 1(1)N \tag{4}$$

$$\frac{d\eta_m(t)}{dt} = f(\eta_m, u_i, h_m) \; \forall \, m = 1(1)M \tag{5}$$

Each ODE of the systems (4, 5), along with current values of the dependent variables as the initial conditions, may be solved by using any of the standard methods to compute the values of the dependent variables. We utilize the classical 4th order Runge-Kutta method. Equations (4, 5) can be rewritten as:

$$\frac{du_i(t)}{dt} = -u_i(t)\frac{(u_{i+1}(t) - u_{i-1}(t))}{2hh} - g\frac{(\eta_{i+1}(t) - \eta_{i-1}(t))}{2hh} \; \forall \, i = 1(1)N \tag{6}$$

$$\frac{d\eta_m(t)}{dt} = u_m(t)\frac{(\eta_{m+1}(t) - \eta_{m-1}(t))}{2hh} - \eta_m \frac{(u_{m+1}(t) - u_{m-1}(t))}{2hh} - 0.2u_m(t) - H(m_i) \; \forall \, m = 1(1)M \tag{7}$$

Thus, we have the system of differential equations of one independent variable t. This system can be easily solved by using fourth order Runge–Kutta scheme.

$$U^{n+1} = U^n + \frac{\Delta t(K_1 + 2K_2 + 2K_3 + K_4)}{6}, \tag{8}$$

Where, $K_1 = F(U^n, \eta^m)$

$$K_2 = F(U^n + \frac{\Delta t}{2}K_1, \eta^m)$$

$$K_3 = F(U^n + \frac{\Delta t}{2}K_2, \eta^m)$$

$$K_4 = F(U^n + \Delta t K_3, \eta^m)$$

$$\eta^{n+1} = \eta^n + \frac{\Delta t(K_1 + 2K_2 + 2K_3 + K_4)}{6}, \tag{9}$$

Where,

$$K_1 = F(\eta^n, U^m, h_m)$$

$$K_2 = F(\eta^n + \frac{\Delta t}{2}K_1, U^m, h_m)$$

$$K_3 = F(\eta^n + \frac{\Delta t}{2}K_2, U^m, h_m)$$

$$K_4 = F(\eta^n + \Delta t K_3, U^m, h_m)$$

As done before we have solved Equation (8) using MOLI then we present a comparison between the MOLI results with the exact solution when $H(x) = d$ as shown in Figure 2. It seems that the absolute errors are small at t = 0.5 and increase with increasing the time as shown in Figures 3 and 4.

Analysis of Tsunami equation using the MOL: Tsunami waves were generated with different wave height and depth at different period. It can be observed that an increase in slope causes a decrease in wave height and when the wave height increase the wave length decrease as shown in Figures 5-14. We not that the period didn't change the properties of the Tsunami wave. Figures 12-14 implies that as the depth of the water decrease the velocity of tsunami increase so in the deep ocean Tsunami have very small amplitude (wave height) and the wave height increase as the depth decrease. Figures 15-20 implies that for a constant height ($H(x) = d$) when the depth increases the velocity decrease and the elevation become constant.

Solution of Tsunami equation using HWM

Haar wavelet method: The Haar wavelet is are a symmetrical capacity of exchanged rectangular waveforms where amplitudes can be unique in relation to one capacity to another and defined over the interim [0,1] as pursues:

$$h_i(x) = \begin{cases} 1, & \varsigma < x \, \beta \\ -1, & \beta < x < \gamma \\ 0 & elsewhere \end{cases} \tag{10}$$

Where $\varsigma = \dfrac{k}{m}$, $\beta = \dfrac{k+0.5}{m}$, and $\gamma = \dfrac{k+1}{m}$

The integer $m = 2^j$ *where* $j = 0, 1, 2, …, J$, denote the wavelet level, and $k = 0, 1, 2, …, m - 1$ is denote the translation

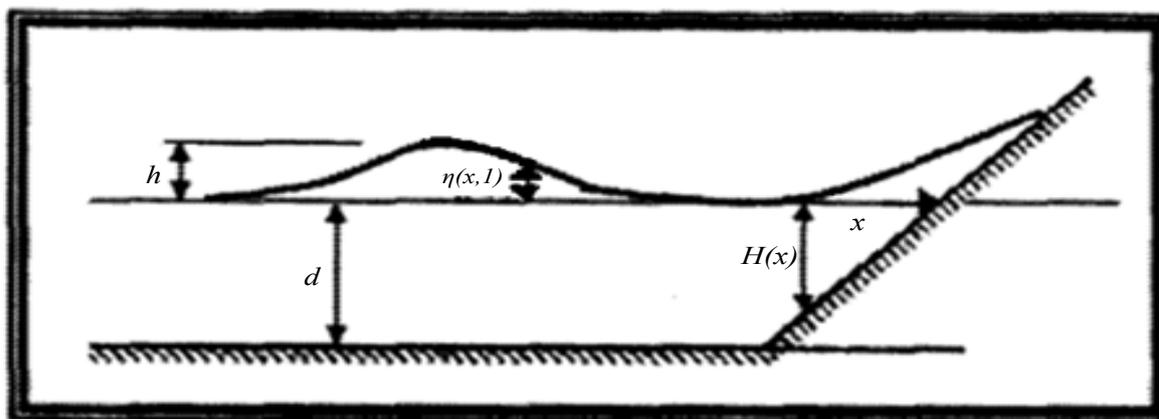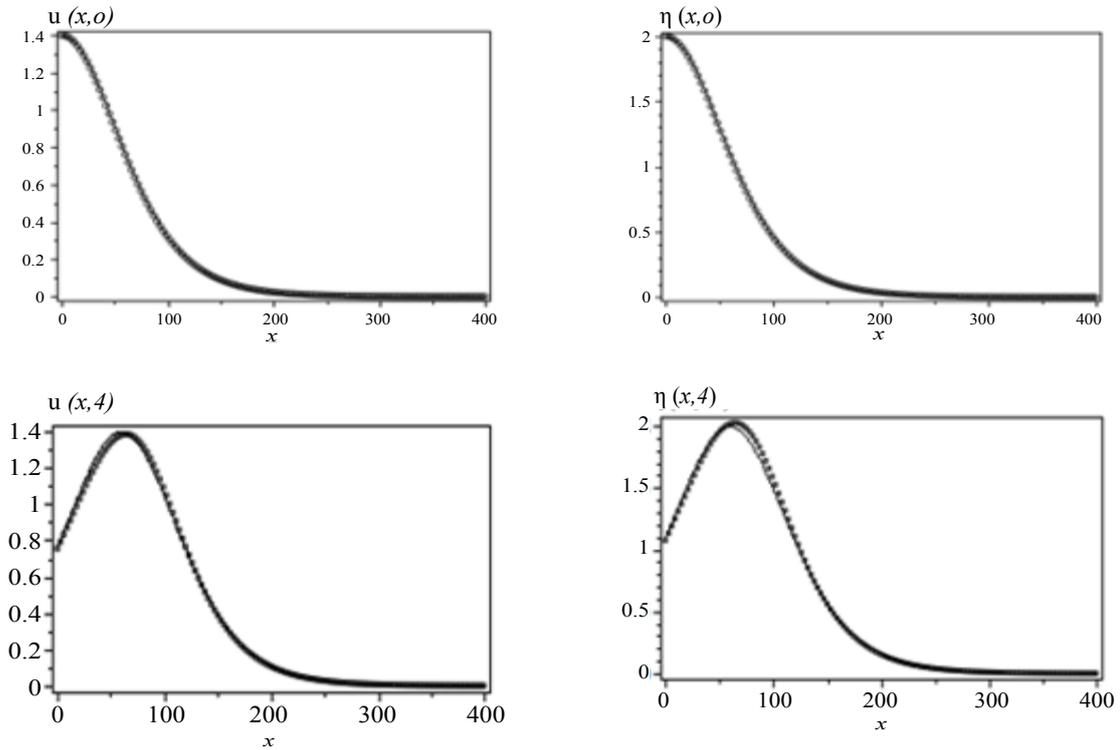***Figure 1.*** *The one-dimensional problem of Tsunami run-up on a shore.*

Figure 2. *Comparison of MOLI (dotted line) and exact (solid line) solutions at N =1600, h=2, H(x)=d=20 and t=0, 0.5, 2 and 4.*
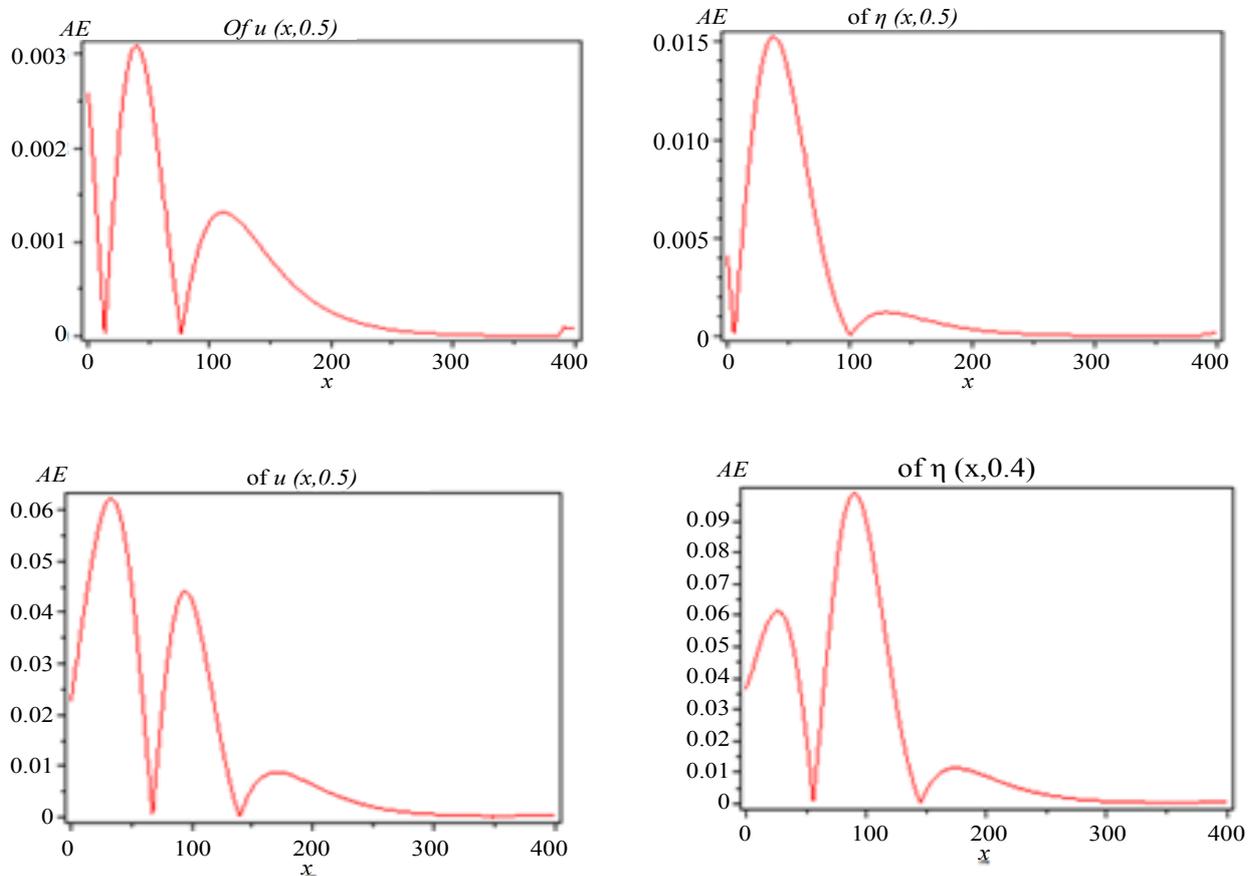

Figure 3. *The absolute error (AE) between the exact solution and the MOLI solution for tsunami equation at what constants at t=0.5, 2 and 4.*
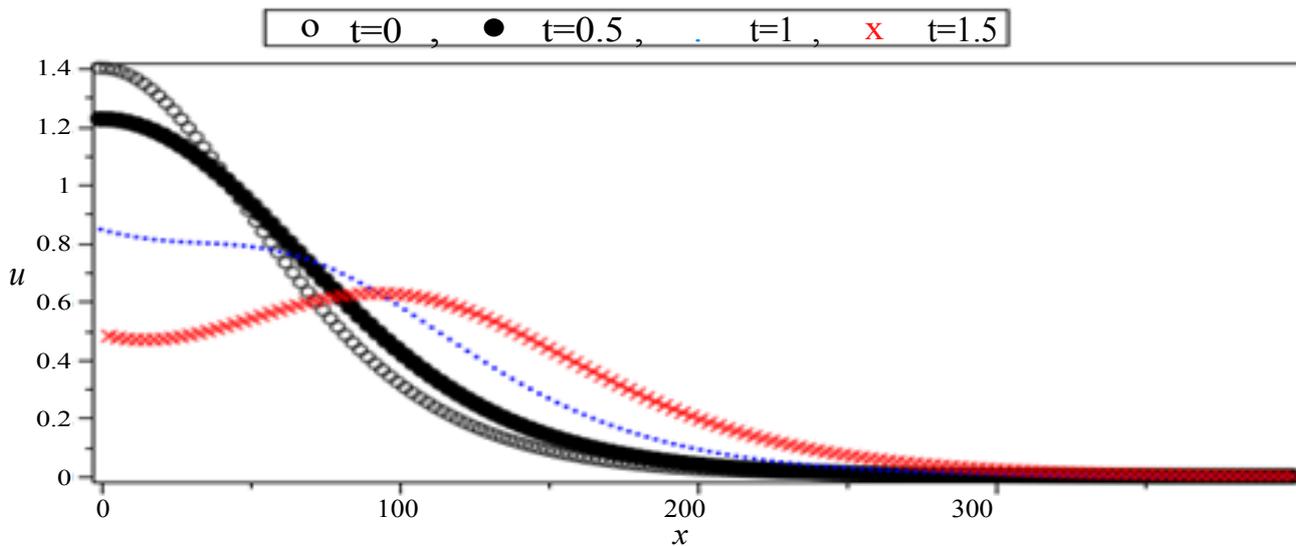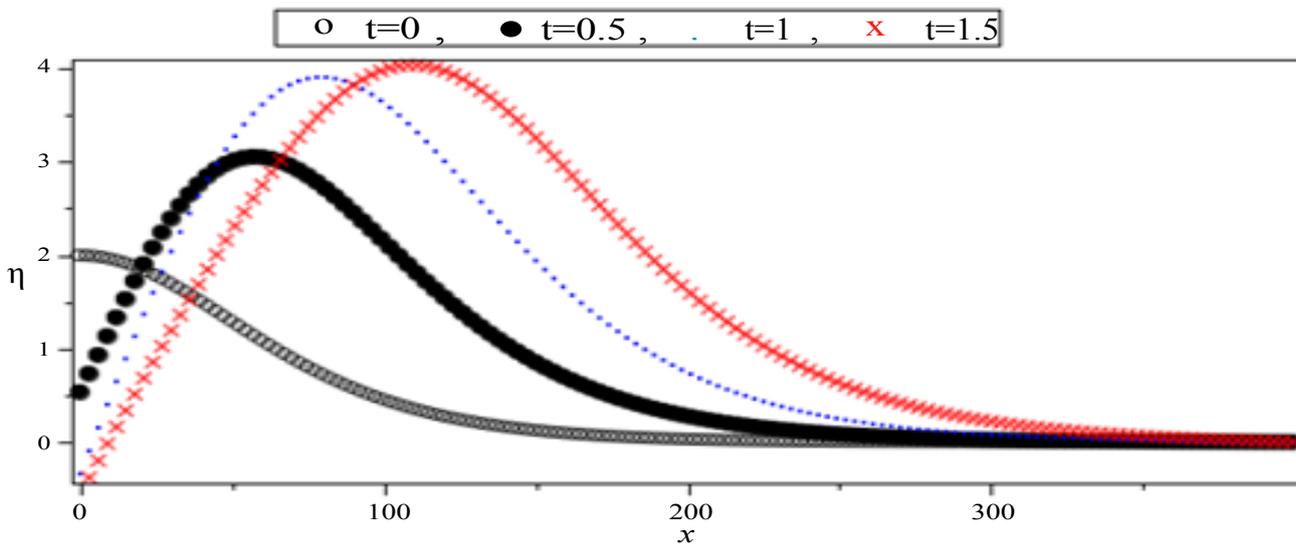
Figure 4. *Graph of velocity u at shore slope 2.*

Figure 5. *Graph of elevation η at shore slope 2.*

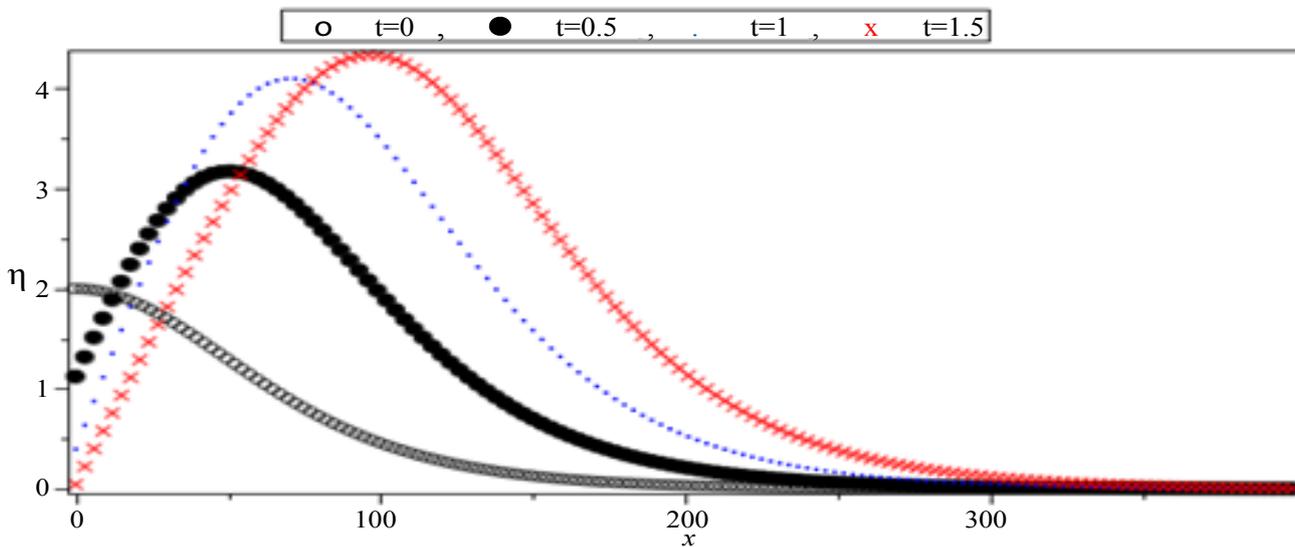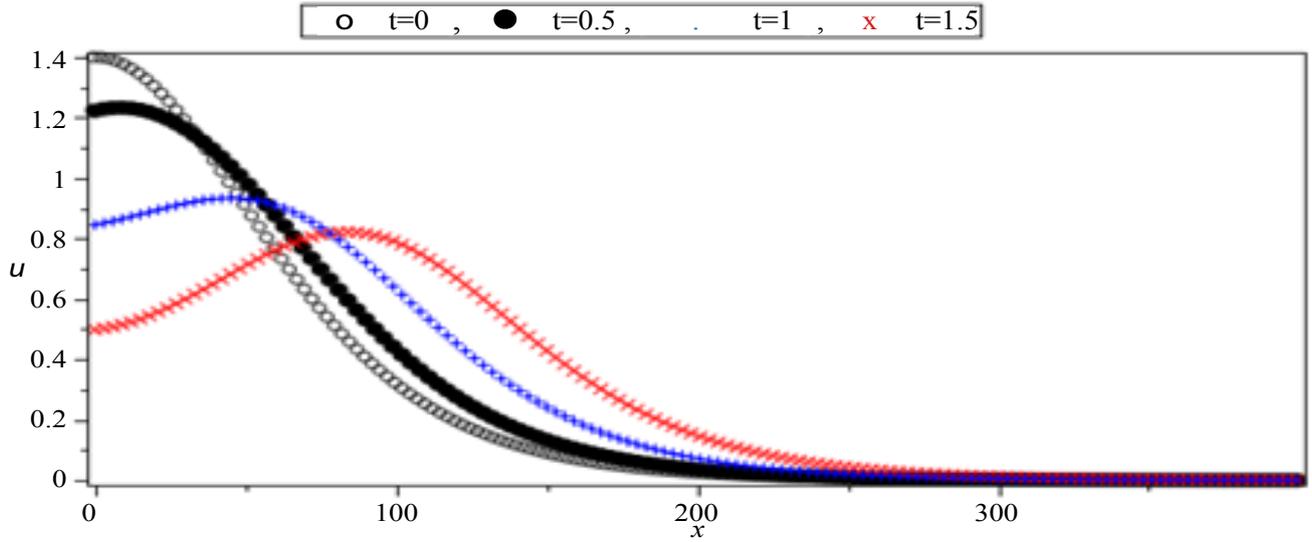**Figure 6.** *Graph of elevation η at shore slope 1.*
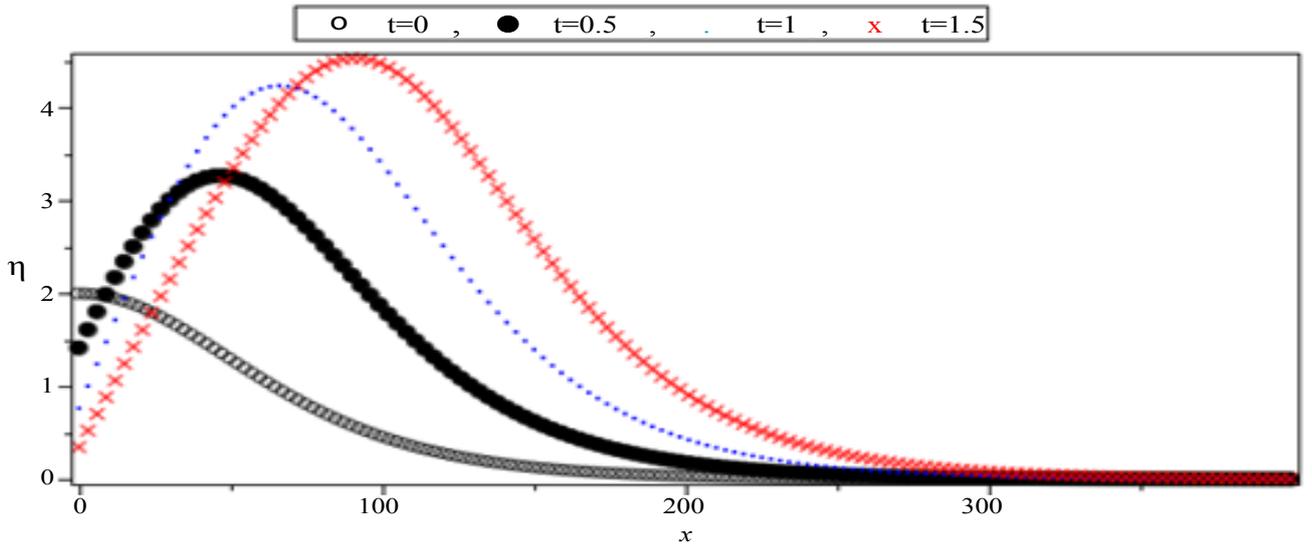
Figure 7. Graph of velocity u at shore slope 0.5.

Figure 8. Graph of elevation η at shore slope 0.5.

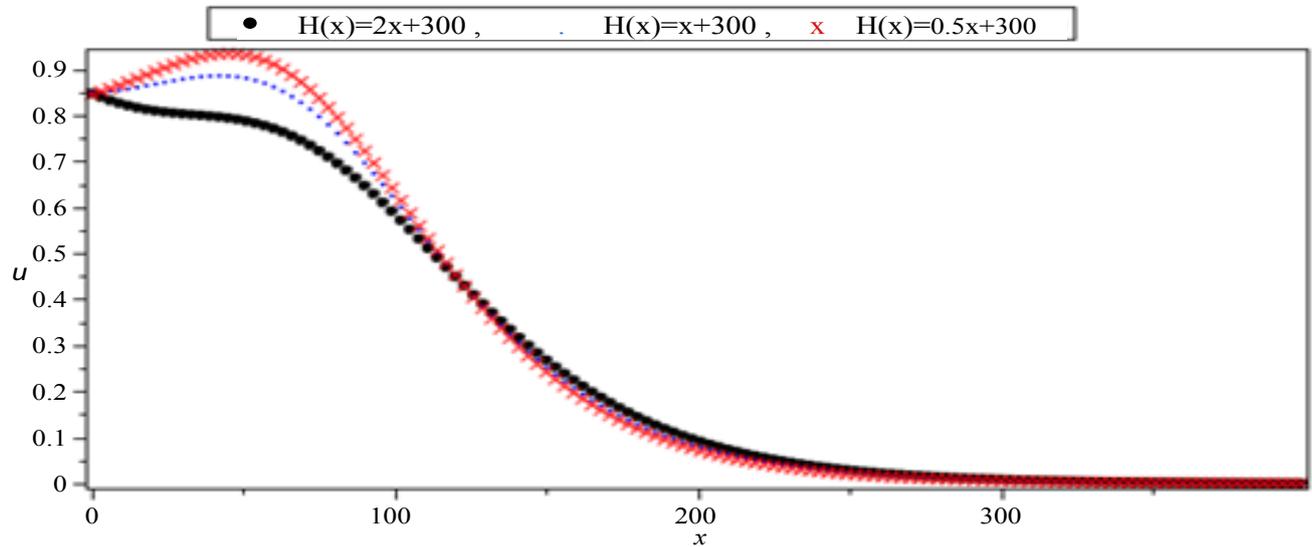*Figure 9. Graph of velocity u at various H(x) and t=1.*

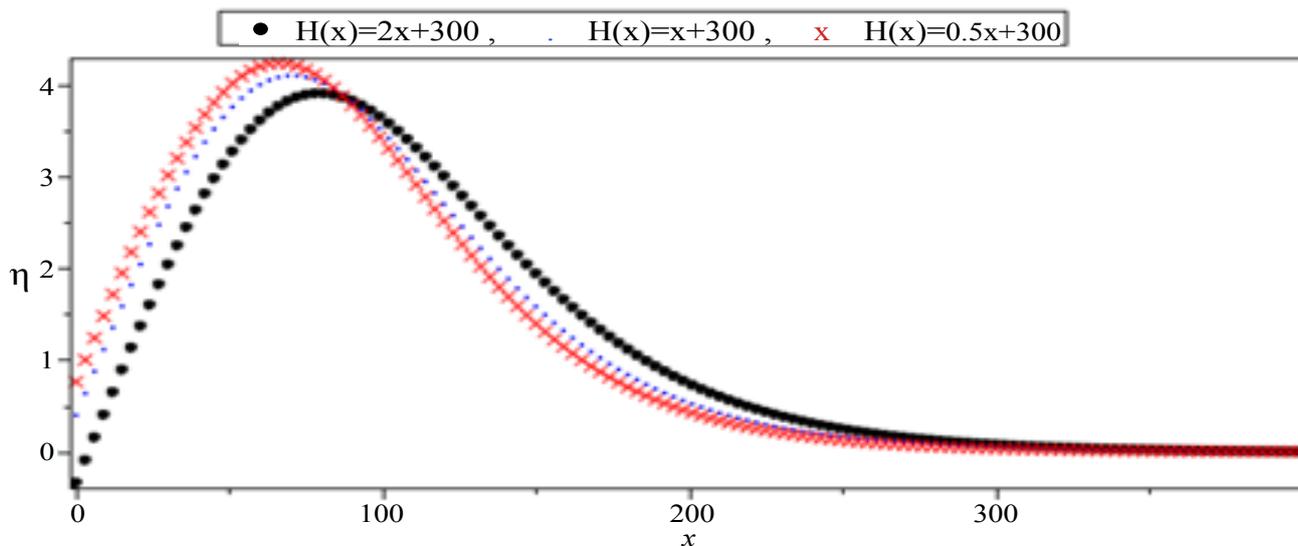**Figure 10.** *Graph of elevation η at various sea depths and t =1.*
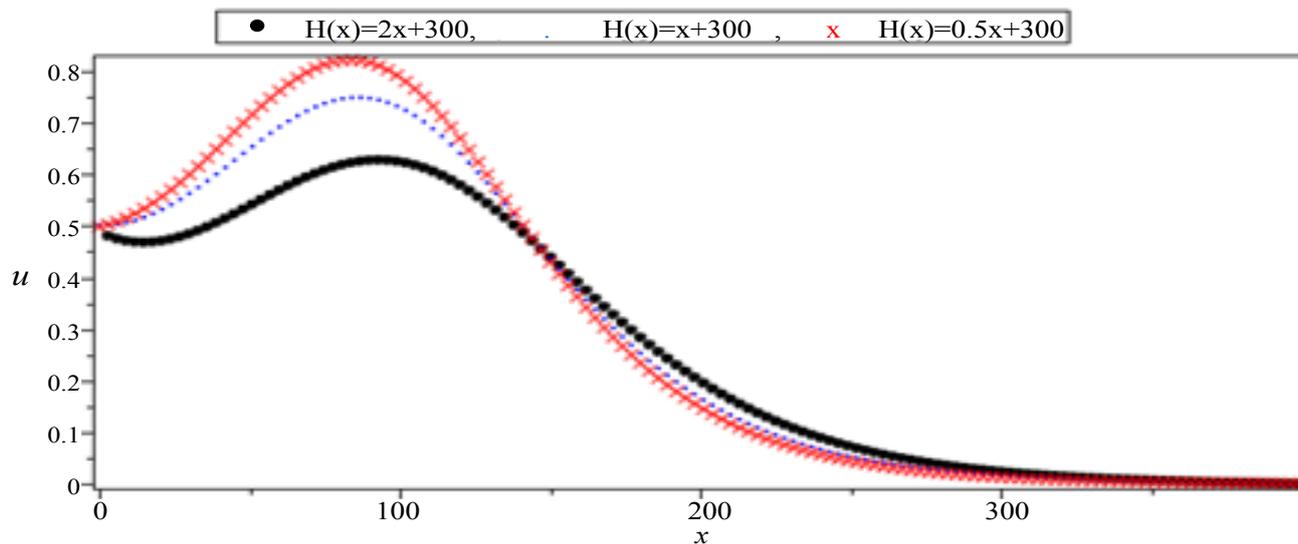
Figure 11. *Graph of velocity u at various sea depths at t =1.5.*
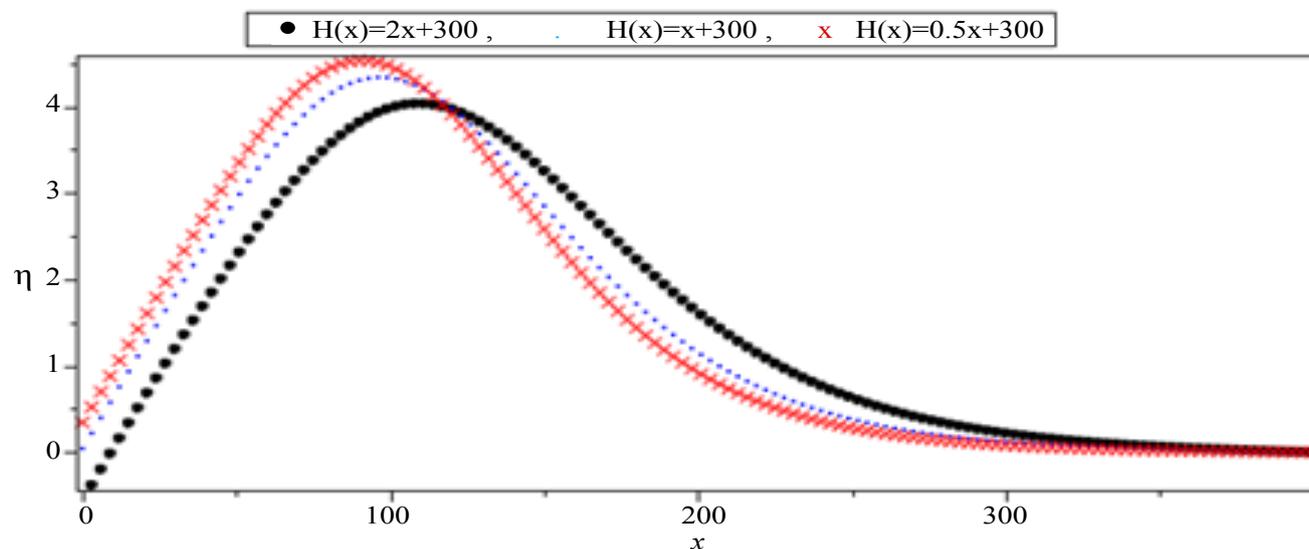
Figure 12. *Graph of elevation η at various sea depths at t =1.5.*
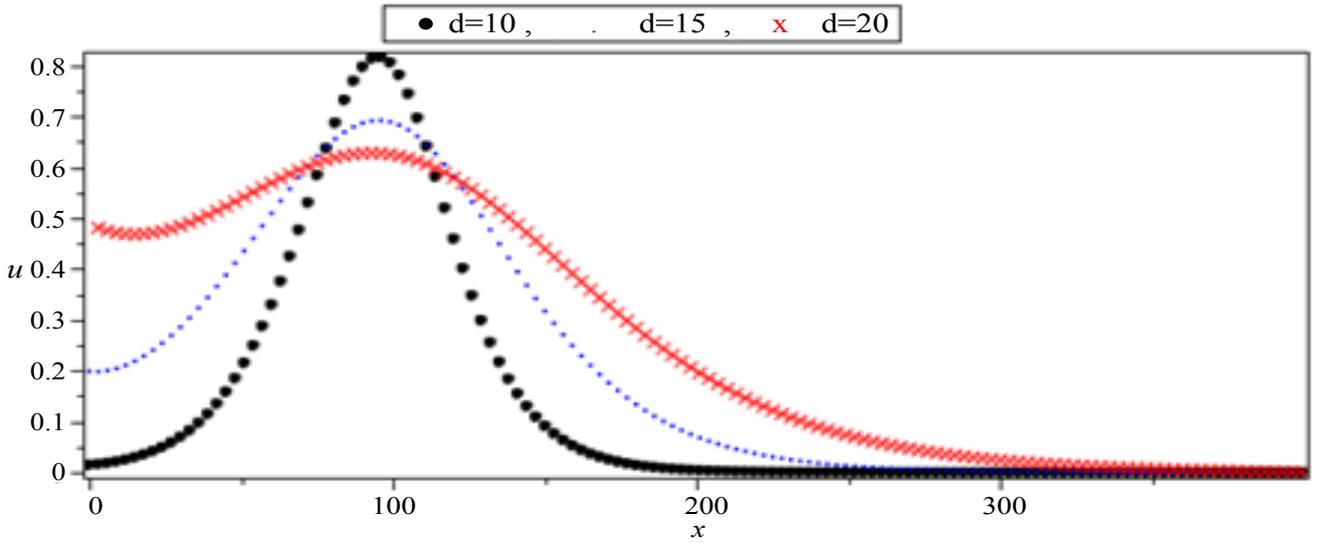
Figure 13. *Graph of velocity u at shore slope 2 and various values of sea depth in open ocean at t =1.5.*
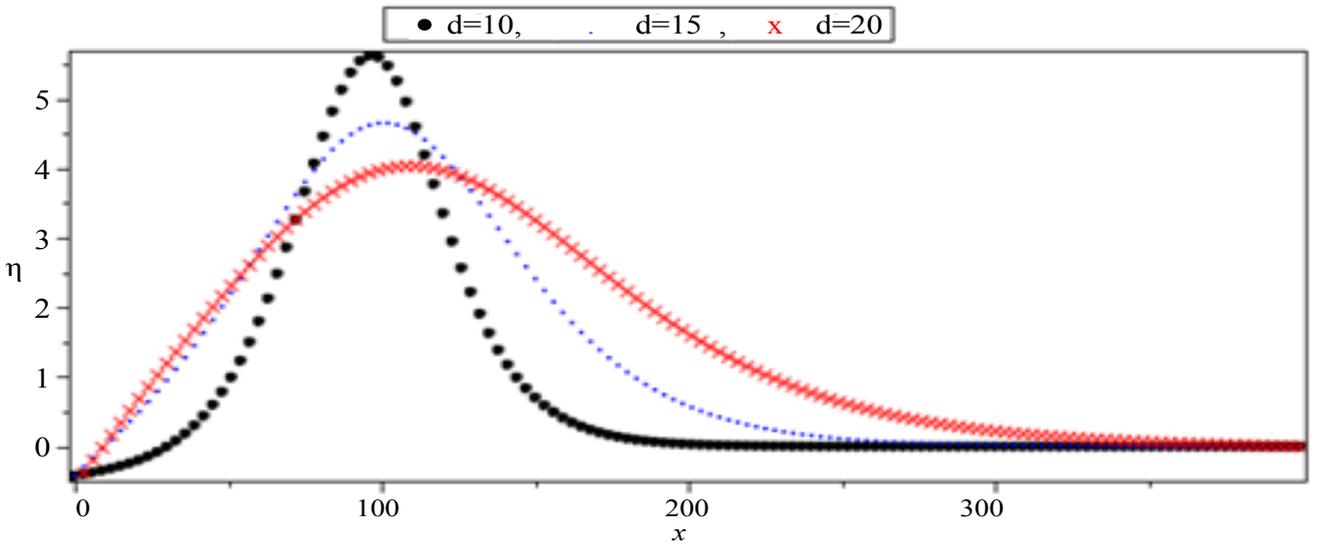
Figure 14. *Graph of velocity u at shore slope 2 and various values of sea depth in open ocean at t =1.5.*

Figure 15. *Graph of elevation η at shore slope 2 and various values of sea depth in open ocean at t =1.5.*

Figure 16. *Graph of velocity u at various values of sea depth in open ocean at h=2 and t=1.5.*

Figure 17. *Graph of elevation η at various values of sea depth in open ocean at h =2 and t =1.5.*

Figure 18. *Graph of elevation η at various values of sea depth in open ocean at h=2 and t=0.*

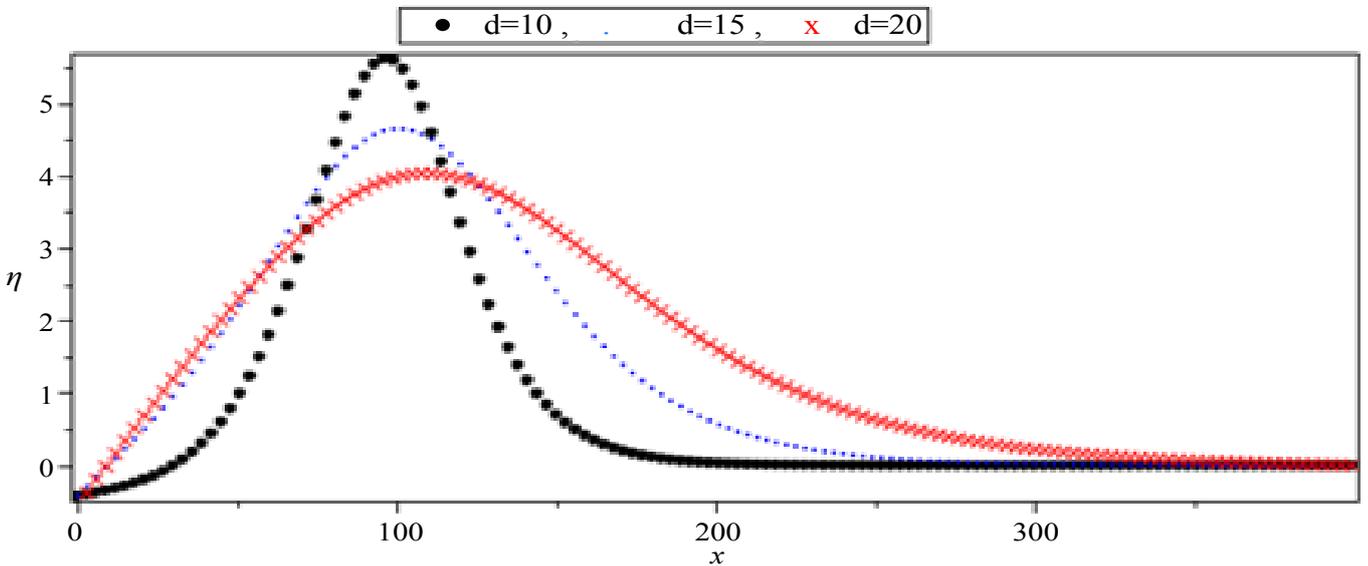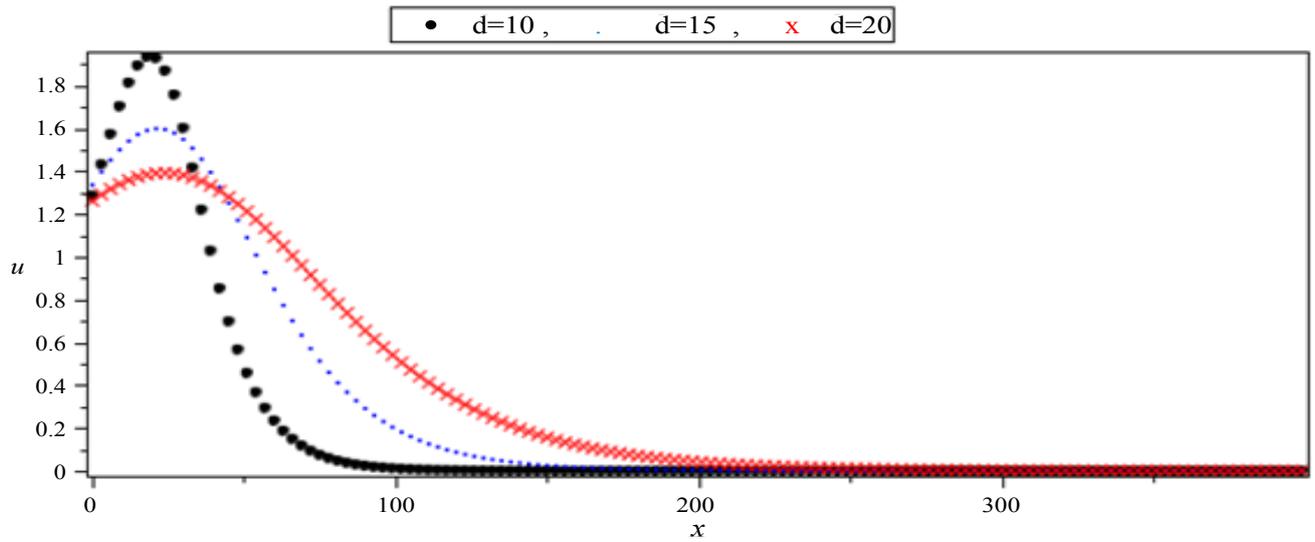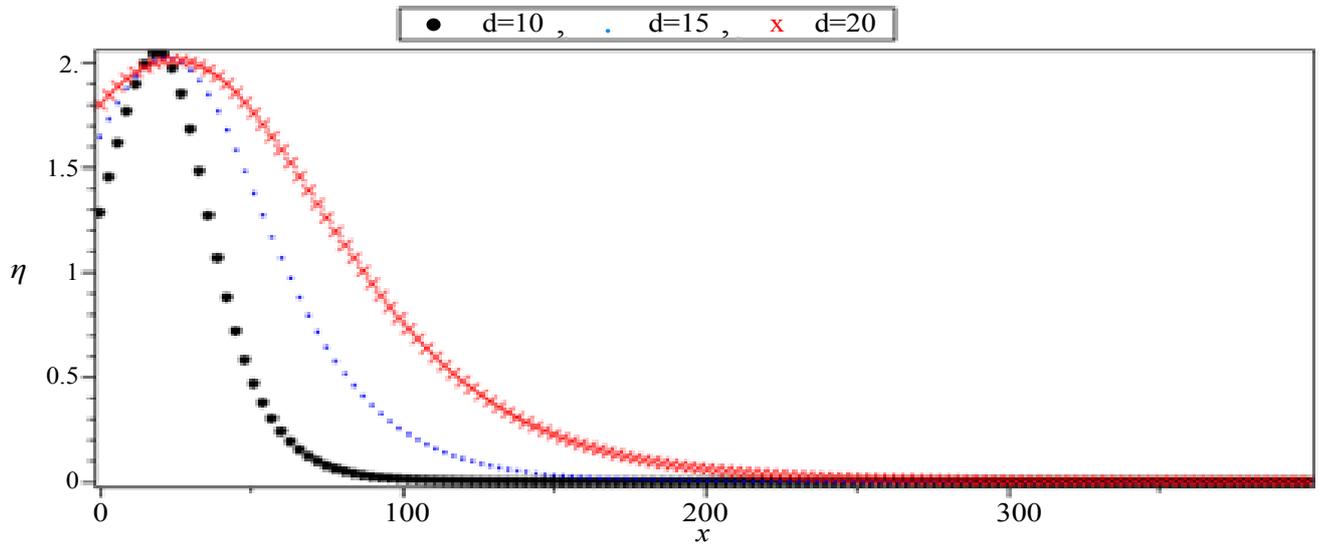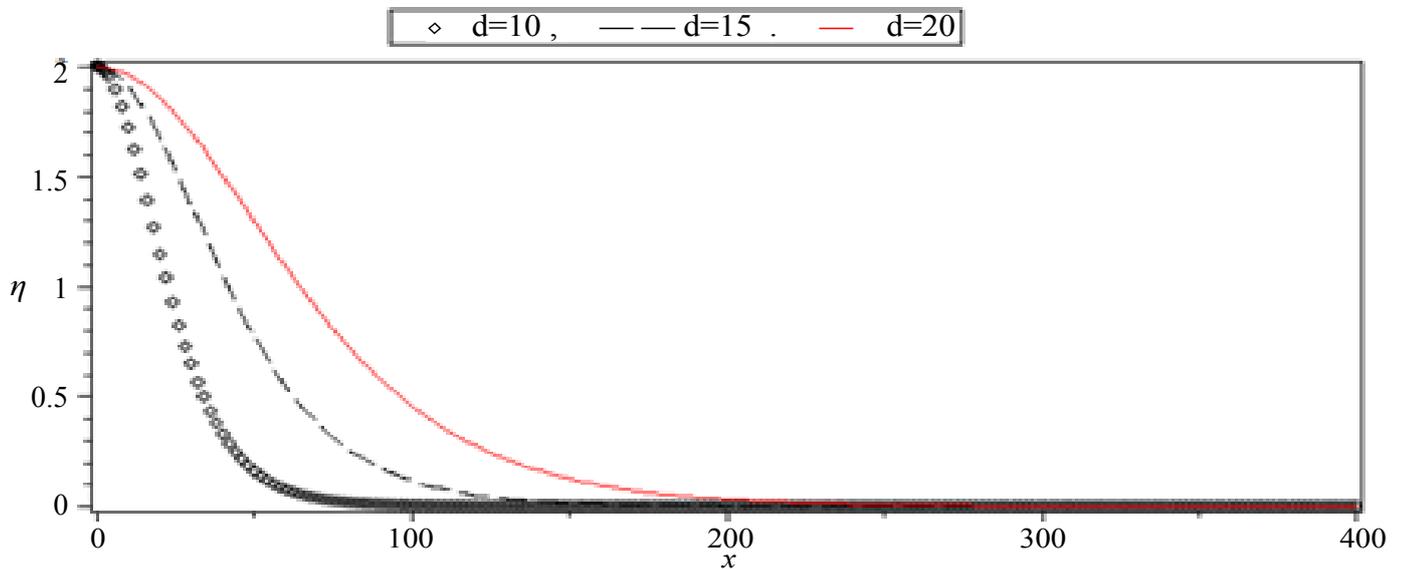***Figure 19.*** *Comparison of MOLI (dotted line) and exact (solid line) solutions of Eq.(1) at N=1600, h=2, d=20 and t =2, 4 and 6 solutions for u.*

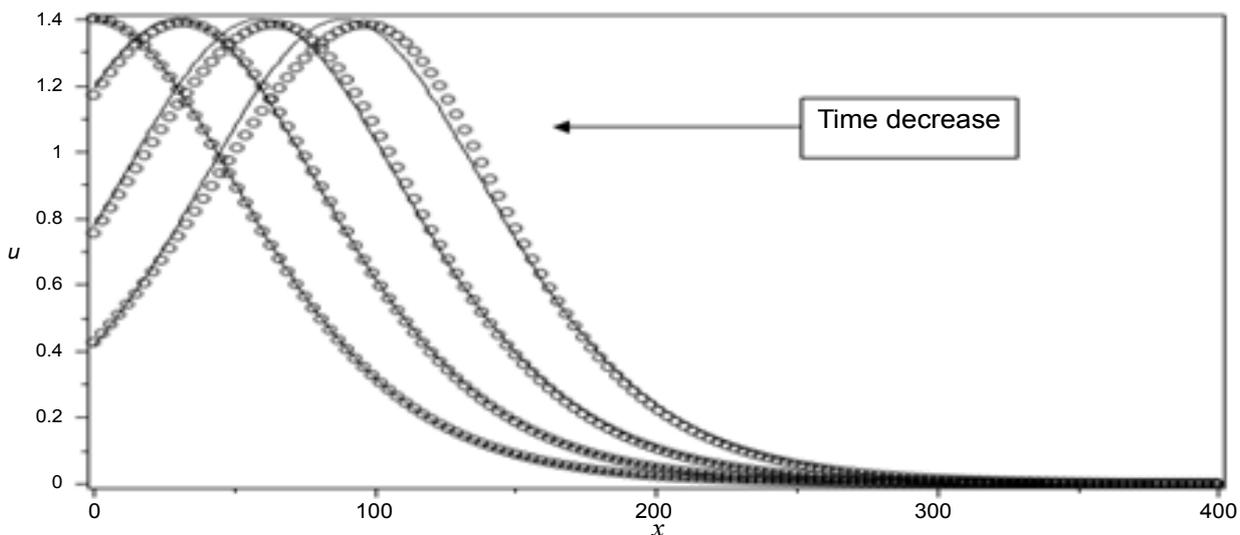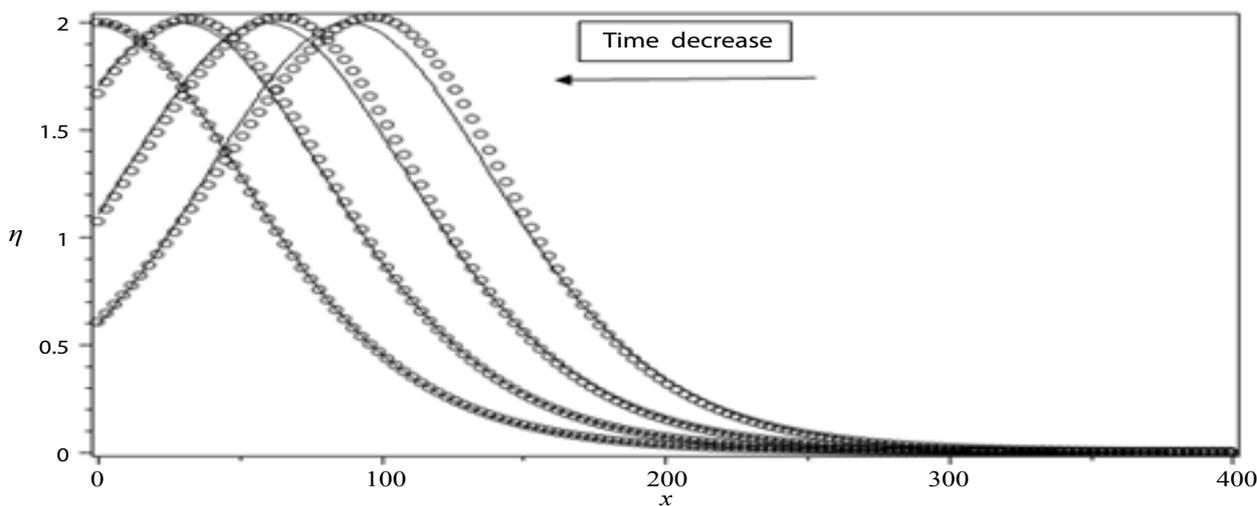***Figure 20.*** *Comparison of MOLI (dotted line) and exact (solid line) solutions of Eq. (1) at N=1600, h=2, d=20 and t =2, 4 and 6 solutions for η.*

parameter. Resolution level known as the integer *J*. The index *i* is established according to the formula $i = m + k +1$. In case of the values $m = 1$, $k = 0$. We own $i = 2$; The value of in is $\mu = 2M = 2^{J+1}$. Assume we define the collocation points $x_l = \frac{l-0.5}{2M}$, with $l = 1, 2, …, 2M$ and the Haar function $h_i(x)$. The Haar coefficient matrix is known as $HAAR(i, l) = h_i(x_l)$ which is a square matrix $2M \times 2M$.

Method of solution: We can assume that $H, L^2[0, 1)$ is a Hilbert space with the inner product defined by $\langle f(x), h \rangle = \int_0^1 f(x) h^T(x) dx$

Since *V* is a finite-dimensional subspace of *H*, it is closed and convex. Thus, for $w \varepsilon H$, there are a unique approximation out of *V* such as g

$$\|w - g\| \le \|g - w\|, \forall h \in V. \tag{11}$$

$$f(x) = \sum_{i=0}^{2M} c_i h_i(x), \tag{12}$$

where the Haar wavelet method coefficients are given by $C_i = \frac{(f(x), h_{(2M)}(x))}{(h_{(2M)}(x), h_{(2M)}(x))}$ such that (.,.) denotes the inner product,

where $\mu = 2M = 2^{j+1}$, *C* and $h(x)$ are $1 \times 2M$ matrices given by:

$$C = [c_1, c_2, c_3, ..., c_{2M}], \ h_{(2M)}(x) = [h_1(x) \ h_2(x) ...h_{2M}(x)] \tag{13}$$

Multiplication of Haar wavelet method: It is important to assess the product of $h_{(\mu)}(x)$ and $h^T_{(\mu)}(x)$, that is called the product matrix of Haar wavelet method. Let

$$M(x) \cong h_\mu(x) h^T_{(\mu)}(x) \tag{14}$$

where $M(x)$ is $2M \times 2M$ matrix. Multiplying the matrix $M(x)$ by vector *C* we obtain

$$M(x)C = \tilde{C} h_{(\mu)}(x) \qquad (15)$$

where $\tilde{C}$ is $2M \times 2M$ matrix and called the coefficient matrix.

Let R is $2M \times 2M$ matrix. Multiplying the matrix R by vector $h_{(\mu)}(x)$ and multiplying the matrix $h_{(\mu)}(x)$ by the resulted matrix $R\, h_{(\mu)}(x)$ we obtain

$$h^T_{(\mu)}(x)\, R\, h_{(\mu)}(x) = \tilde{R}\, h_{(\mu)}(x) \qquad (16)$$

where $\tilde{R}$ is $1 \times 2M$ matrix and called the coefficient matrix with the powerful properties of Eq. (16) We can achieve $\tilde{R}$ by away like $\tilde{c}$ we can convert the Volterra part of integral and Integro-Differential Equations System equations to an algebraic equation.

HWM applied to the Tsunami run-up model yields an analytical solution in terms of rapidly convergent easily computable terms.

Equation (1) written as:

$$u_t + u u_x + g\, \eta_x = 0$$

$$\eta_t + u\, \eta_x + H_x u + \eta u_x + H u_x = 0 \qquad (17)$$

We define the linear operator L_t as: $L_t = \dfrac{\partial}{\partial t}$.

We consider the application of the HWM to the model of tsunami Run-up on the coast with the initial conditions. Where the initial wave height *(h)* and the constant value of sea depth *(d)* are given by $h = 2$ and $d = 20$. And we consider three different cases of shore slopes with the depth function $H(x)$ given by

Case I: Shore slope = 0.5, $H(x) = 0.5x + 300$

Case II: Shore slope = 1, $H(x) = x + 300$

Case III: Shore slope = 2, $H(x) = 2x + 300$

For the Tsunami model we have computed analytic solutions to equation (15) using Maple 15 (Tables 1A-1H).

Table 1A. *The values of velocity u at shore slope 2, d=10 and h=2 at t=0.5.*

x	MOL	HWM
0	0.775435705843	0.7754895431117
15	0.927626747086	0.9278187550637
30	1.064755315418	1.0647366440529
60	0.361910033228	0.361897456040
75	0.138698696189	0.138739288119
90	0.049090131969	0.049112586589
105	0.016875068044	0.016884077653
120	0.005735339986	0.005738636481
135	0.001939263041	0.001940430796
150	0.000653791689	0.0006541999083

Table 1B. The values of elevation η at shore slope 2, d=10 and h=2 at t=0.5.

x	MOL	HWM
0	0.024695547836	0.0245922643554
15	3.726959266545	3.7285142696089
30	5.843373800096	5.8435239641855
45	4.617805767048	4.6162585617936
75	0.914469655265	0.9147581475193
90	0.33533007619	0.3354912710164
105	0.11910092844	0.1191671460289
120	0.04173323531	0.0417580771231
135	0.01452155544	0.0145305745015
165	0.00173613581	0.0017372827749

Table 1C. The values of velocity u at shore slope 2, d=10 and h=2 at t=1.5.

x	MOL	HWM
0	0.0146767645	0.0146561
15	0.0295304605	0.0295489
30	0.0695353242	0.06953156
45	0.1582343262	0.15823748
60	0.3297004405	0.3297049
75	0.58442395994	0.58442849
90	0.79737141212	0.79737165
105	0.74439222240	0.74643916
120	0.459331720102	0.41615933
135	0.217011889604	0.21701489
150	0.091314130163	0.0913489

Table 1D. The values of elevation η at shore slope 2, d=10 and h=2 at t=1.5.

x	MOL	HWM
0	-0.42024584	-0.4208156232
15	-0.28024021	-0.2808516240
30	-0.00319321	-0.0038516193
45	0.600639997	0.600684563
60	1.800691310	1.800965691
75	3.667549880	3.667541649
90	5.378742395	5.378715423
105	5.256640812	5.25664084
120	3.354546155	3.35454566
135	1.630955322	1.63095856
150	0.704718125	0.704714815
165	0.290193924	0.29019392
180	0.116892752	0.11689275
195	0.046519873	0.04658955
210	0.018365262	0.01836562

Table 1E. The values of velocity u at shore slope 0.5, d=20 and h=2 at t=0.5.

x	MOL	HWM
0	1.22269209769	1.222708384879
15	1.226318884710	1.226331637389
30	1.163123922891	1.163127093566
45	1.0399381742546	1.03993184859
60	0.8761100530607	0.87609944751
75	0.6984085017272	0.69839912176
90	0.5310853501003	0.53107967707
105	0.3889804809915	0.38897825424
120	0.2769674718309	0.276967397910
135	0.1932186907370	0.193219595725
150	0.1328632088968	0.132864370631
165	0.090451826012	0.090452901556
180	0.061158023987	0.0611588959021
195	0.041158981836	0.0411596409561
210	0.027612428004	0.0276129062761

Table 1F. The value of elevation η at shore slope 0.5, d=20 and h=2 at t=0.5.

x	MOL	HWM
0	1.4124775362	1.4124936789
15	2.3416193724	2.3416204856
30	3.0069417436	3.0069163387
45	3.2648281925	3.2647767328
60	3.1256293454	3.1255710977
75	2.71598934708	2.7159458972
90	2.19289876937	2.1928784260
105	1.67726333464	1.6772618619
120	1.23401098686	1.2340199796
135	0.88345785062	0.8834704292
150	0.62064377369	0.6206560071
165	0.43038827925	0.4303985338
180	0.29581699704	0.2958249364
195	0.20209333266	0.2020991967
210	0.13749195049	0.1374961563

Table 1G. The values of velocity u at shore slope 0.5, d=20 and h=2 at t=1.5.

x	MOL	HWM
0	0.4985818076	0.4985836857323
15	0.5336565233	0.5336615432408
30	0.5996877272	0.5997006075876
45	0.6827276484	0.6827487558092
60	0.7618775671	0.7619008384469
75	0.8131028471	0.8131169847658
90	0.8166623672	0.8166577866131
105	0.7657027201	0.7656801563363
120	0.6699689574	0.6699396773769
135	0.5505504249	0.5505265903356
150	0.4294116742	0.4293984317104
165	0.3216311753	0.3216271428251
180	0.2337983357	0.2337998131882
195	0.1663473317	0.1663511371678
210	0.1165867115	0.1165909342995

Table 1H. The values of elevation η at shore slope 0.5, d=20 and h=2 at t=1.5.

x	MOL	HWM
0	0.34470688975	0.34470362511
15	1.24270022801	1.24275101653
30	2.15149435851	2.15160099089
45	3.02918112501	3.02933470122
60	3.79244305608	3.79260565183
75	4.327893886213	4.32799903195
90	4.533622004821	4.53361205666
105	4.373915609464	4.37379323263
120	3.908168808445	3.90800089538
135	3.265244937672	3.26510596417
150	2.582656116358	2.58257922024
165	1.958599972169	1.95857829096
180	1.440121789292	1.44013349656
195	1.035770710691	1.03579653611
210	0.733481265177	0.73350945764

Discussion and Conclusion

The MOL and HWM for approximating arrangements of Non-Linear Tsunami Model of Coupled Partial Differential Equations. The outcomes announced here give additional proof of the convenience of MOL. The MOL was unmistakably extremely effective and ground-breaking strategy in finding the arrangements of the proposed coupled nonlinear conditions. It is significant that to expand the exactness of the HWM the precision of the arrangement. This prompt increments the many-sided quality of computations. Be that as it may, the MOL is steady, easy to execute and requires moderately little computational assets. The numerical technique created in this investigation to anticipate the run-up of tallness waves gives a basic yet sensibly great forecast of different parts of the run-up process. The outcomes concur well with the exploratory information [9]. From this investigation we see that the technique for lines (MOL) is an appropriate strategy for the examination of long wave marvels started in the shallow water area. Besides, correlations with standard HWM arrangements uncover that the MOL gives a compelling calculation to mimicking shallow water conditions [10-15].

References

1. Hayashi Y, Munemoto K, Imamura F. Effect of the Emperor seamounts on trans-oceanic propagation of the 2006 Kuril Island earthquake Tsunami. Geophys Res Lett. 2008;35:611-22.

2. Koshimura, Imamura F, Shuto N. Characteristics of Tsunamis propagating over oceanic ridges. Journal of Physics. 2001;24:213-29.

3. Kowalik. Basic Relations between Tsunami calculation and their physics. Science of Tsunami Hazards. 2003;21:154-73.

4. Kowalik Z, Knight W, Logan T, et al. Numerical modelling of the Indian Ocean tsunami. Science of Tsunami Hazards. 2007;17:97-122.

5. Kowalik Z, Knight W, Logan T, et al. Numerical modelling of the global tsunami. Science of Tsunami Hazards. 2005;23:40- 56.

6. Marchuk AG, Anisimov AA. A method for numerical modeling of Tsunami run-up on the coast of an arbitrary profile. International Tsunami Symposium. 2001;7:7-27.

7. Peregrine DH, Williams SM. Swash overtopping a truncated plane beach. J Fluid Mech. 2001;440:391-99.

8. Jacobson T. Vorticity on a Barred Beach. J Fluid Mech. 2001;449(1):313-39.

9. Gedik N, Irtem E, Kabdasli S. Laboratory investigation on Tsunami run-up. J Ocean Eng. 2005;32:513–28.

10. Ramadan MA, Ali MR. An efficient hybrid method for solving fredholm integral equations using triangular functions. New Trends in Mathematical Sciences. 2017;5(1):213-24.

11. Sadat RE, Ali MR. Application of the method of lines for solving the KdV-Burger equation. Journal of Abstract and Computational Mathematics. 2017;2(2):39-51.

12. Ramadan MA, Ali MR. Application of Bernoulli wavelet method for numerical solution of fuzzy linear Volterra-Fredholm integral equations. Communication in Mathematical Modeling and Applications. 2017;2(3):40-9.

13. Ali MR. Approximate solutions for fuzzy Volterra integro-differential equations. Journal of Abstract and Computational Mathematics. 2018;3(2):11-22.

14. Sadat R, Kassem M. Explicit solutions for the (2+ 1)-dimensional Jaulent–Miodek equation using the integrating factors method in an unbounded domain. Math Comput Appl. 2018;23(1):1-9.

15. Ramadan MA, Ali MR. Solution of integral and Integro-Differential equations system using Hybrid orthonormal Bernstein and block-pulse functions. Journal of Abstract and Computational Mathematics. 2017;2(1):35-48.

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