A new approach for solving linear equations with first order through derivatives.

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Abstract

This paper proposes a simple method to solve the first order linear equations, the proposed method is equivalent to classical Cramer's rule for solving general systems of 2 linear equations, then it describes if there is a relationship between this method and the derivatives. The results show that there is a possible relationship between the method presented in this paper and the derivatives. Furthermore, we can use the first derivative to solve linear equations with first order.

Keywords: Linear equations, Matrix, First derivatives, Cramer's rule.

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Introduction

There are various methods to solve the linear equation, The Cramer's rule is the most common of these methods [1], Klein [2] described the approach based upon Cramer's rule, the of the linear equation system can be written in matrix form: $\mathrm{A}x=b$, Cramer's rule is efficient in solving systems of 2 linear equations. Some recent developments of using Cramer's rule described in some papers, these papers can be found in [3-5] and the references therein.

Solving linear equations

First, this paper introduces a simple method for solving general systems of 2 linear equations, and we will prove it using Cramer's rule as following:

Rule (1): If we have a real numbers a_1, a_2, b_1, b_2, c_1 and c_2, and the variables x, y, if we have the following system with first order linear equation:

$a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ and the determinant $D=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$

Then:-

$$D^2x=\begin{vmatrix} -(a_1c_2+a_2c_1) & a_1b_2+a_2b_1 \\ -(b_1c_2+b_2c_1) & 2b_1b_2 \end{vmatrix}$$

$$D^2y=\begin{vmatrix} 2a_1a_2 & -(a_1c_2+a_2c_1) \\ a_1b_2+a_2b_1 & -(b_1c_2+b_2c_1) \end{vmatrix}$$

Proof: We can prove the above rule by using Cramer's rule, as we know when we use Cramer's rule we find that:-

$$x=\frac{b_2c_1-b_1c_2}{a_1b_2-a_2b_1} \tag{1}$$

and $$y=\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1} \tag{2}$$

First, we want to prove that $D^2x=\begin{vmatrix} -(a_1c_2+a_2c_1) & a_1b_2+a_2b_1 \\ -(b_1c_2+b_2c_1) & 2b_1b_2 \end{vmatrix}$

$$\Rightarrow (a_1b_2-a_2b_1)(a_1b_2-a_2b_1)x=-2a_1c_2b_1b_2-2a_2c_1b_1b_2+a_1b_2\,b_1c_2+a_1b_2^2\,c_1+a_2b_1^2\,c_2+a_2b_1\,b_2c_1$$

$$\Rightarrow (a_1b_2-a_2b_1)(a_1b_2-a_2b_1)x=-2a_1c_2b_1b_2-2a_2c_1b_1b_2+a_1b_2\,b_1c_2+a_1b_2^2\,c_1+a_2b_1^2\,c_2+a_2b_1\,b_2c_1$$

$$\Rightarrow (a_1b_2-a_2b_1)(a_1b_2-a_2b_1)x=-a_1c_2b_1b_2+a_1b_2^2\,c_1-a_2c_1b_1b_2+a_2b_1^2\,c_2$$

$$\Rightarrow (a_1b_2-a_2b_1)(a_1b_2-a_2b_1)x=-a_1c_2b_1b_2+a_1b_2^2\,c_1-a_2c_1b_1b_2+a_2b_1^2\,c_2$$

$$\Rightarrow (a_1b_2 - a_2b_1)(a_1b_2 - a_2b_1)x = a_1b_2(b_2c_1 - b_1c_2) - a_2b_1(b_2c_1 - b_1c_2)$$
$$\Rightarrow (a_1b_2 - a_2b_1)(a_1b_2 - a_2b_1)x = (b_2c_1 - b_1c_2) - (a_1b_2 - a_2b_1)$$
$$\Rightarrow x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

Similarly, we can prove also that $y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$ by using the above method.

The possible relation between rule 1 and the first derivative: -

Now we will discuss if there is a relation between rule 1 and derivatives: -

Using the same equation in rule (1):-

$$\left.\begin{matrix} a_1x + b_1y = c_1 \Rightarrow a_1x + b_1y - c_1 = 0 \\ a_2x + b_2y = c_2 \Rightarrow a_2x + b_2y - c_2 = 0 \end{matrix}\right\} f(x,y) = (a_1x + b_1y - c_1)(a_2x + b_2y - c_2)$$

$$\frac{df}{dx} = (2a_1a_2)x + (a_1b_2 + a_2b_1)y - (a_1c_2 + a_2c_1)$$

$$\frac{df}{dy} = (a_1b_2 + a_2b_1)x + (2b_1b_2)y - (b_1c_2 + b_2c_1)$$

We can represent $\frac{df}{dx}$ and $\frac{df}{dy}$ by the following matrix:-

$$\begin{matrix} \textit{Coefficients of } x & \textit{Coefficients of } y & \textit{Coefficients of the constants} \\ \downarrow & \downarrow & \downarrow \end{matrix}$$
$$\begin{bmatrix} 2a_1a_2 & a_1b_2 + a_2b_1 & -(a_1c_2 + a_2c_1) \\ a_1b_2 + a_2b_1 & 2b_1b_2 & -(b_1c_2 + b_2c_1) \end{bmatrix}$$

Thus, if we want to find the values of x and y we can easily reach to the same results in Rule 1, where the two columns that we used to find x in rule 1 is similar to the coefficients of the constants and the variable y in the matrix of $\frac{df}{dx}$ respectively:-

$$D^2x = \begin{vmatrix} -(a_1c_2 + a_2c_1) & a_1b_2 + a_2b_1 \\ -(b_1c_2 + b_2c_1) & 2b_1b_2 \end{vmatrix}$$

Similarly, the two columns that we used to find y in rule 1 is similar to the coefficients of the variable x and the constants in the matrix of $\frac{df}{dy}$ respectively: -

$$D^2y = \begin{vmatrix} 2a_1a_2 & -(a_1c_2 + a_2c_1) \\ a_1b_2 + a_2b_1 & -(b_1c_2 + b_2c_1) \end{vmatrix}$$

Solving linear equations with first order by first derivatives

To explain how to solve linear equations with first order by first derivatives; suppose we have the following linear equation system: -

$$x + 2y = 4$$
$$3x - y = 5$$

Let $f(x,y) = (a_1x + b_1y - c_1)(a_2x + b_2y - c_2) = (x + 2y - 4)(3x - y - 5)$

$$\frac{df}{dx}=1(3x-y-5)+3(x+2y-4)\Rightarrow\frac{df}{dx}=6x+5y-17$$

$$\frac{df}{dy}=2(3x-y-5)-1(x+2y-4)\Rightarrow\frac{df}{dy}=5x-4y-6$$

$$D=\begin{vmatrix}1 & 2\\ 3 & -1\end{vmatrix}=-7$$

$$D^2x=\begin{vmatrix}-(a_1c_2+a_2c_1) & a_1b_2+a_2b_1\\ -(b_1c_2+b_2c_1) & 2b_1b_2\end{vmatrix}$$

$$(-7)^2x=\begin{vmatrix}-17 & 5\\ -6 & -4\end{vmatrix}$$

$$49x=98\Rightarrow x=2$$

Now to find the value of Y:-

$$D^2y=\begin{vmatrix}2a_1a_2 & -(a_1c_2+a_2c_1)\\ a_1b_2+a_2b_1 & -(b_1c_2+b_2c_1)\end{vmatrix}$$

$$(-7)^2y=\begin{vmatrix}6 & -17\\ 5 & -6\end{vmatrix}$$

$$49y=49\Rightarrow y=1$$

Conclusion

We have studied a simple method for solving systems of 2 linear equations. The method can be easily applied to systems of 2 linear equations. Also, we have described if there is a relationship between this method and the first derivative, the paper show that there is a possible relationship between them, and we can solve linear equations with first order by first derivatives.

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