TESTING THE DIFFERENTIAL EFFECT OF A MATHEMATICAL BACKGROUND ON STATISTICS COURSE PERFORMANCE: AN APPLICATION OF THE CHOW-TEST

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ABSTRACT

This paper focuses on understanding the differential effect of students' mathematical background (prerequisite) knowledge on Statistics course. Introductory Statistics is one of the required courses for business and economics majors. Students can choose one of several mathematics based prerequisite courses to gain necessary background knowledge for the Statistics course. Among several possible prerequisite courses, we considered only two different calculus courses as background knowledge for Statistics course to compare; namely Applied Calculus and Calculus-I. Students' success on subsequent course is greatly affected by the prerequisite courses taken by students. Mathematical topics vary widely among these courses providing different breadth of background knowledge to prepare students for the Statistics course. Therefore, the objective of this research is to observe the significance and magnitude of differential effect of two different calculus courses on the Statistics course performance.

Chow test is being applied through regression models provided consistent conclusions about the significance and differential effect of mathematical background knowledge on the performance of Statistics course. Specifically, we have found that students who took the Calculus-I received higher grades on average in Statistics course than did students who took Applied Calculus. Thus, students with added traditional calculus orientation do have greater statistical proficiency. Furthermore, the analysis also reveals that students' are situated in an advantageous position when taking Calculus-I than with Applied Calculus.

INTRODUCTION

Identifying appropriate prerequisite course is a key ingredient in designing the optimum curriculum program. An academic advisor's primary challenge is to match students' background knowledge with the courses they are taking. Identifying the most suitable course among the several available alternative prerequisite courses to meet students' need is a source of continuous debate among the academicians. This paper addresses the issue of students with different mathematical background perform differently in Statistics course. Higgins (1999) recognized that statistical reasoning should be considered an important component of any undergraduate program. Further discussion on statistical reasoning can be found in Garfield (2002) and DelMas et. al.(1999). Several different factors may affect students' performance (Dale & Crawford, 2000) in a course, including students' background knowledge. Therefore, understanding (Choudhury, Hubata & St. Louis, 1999) and acquiring the proper background knowledge is the primary driver of success (Bagamery, Lasik & Nixon, 2005; Sale, Cheek & Hatfield, 1999).

Students' performance (Trine & Schellenger, 1999) in a course is primarily affected by the prerequisite courses taken that fabricate their background knowledge. Because of their diverse level of preparedness and accumulated background knowledge that builds their long-term human capital, differential effect that is attributable to different perquisite courses can be evaluated through students' performance on subsequent courses. Literatures in this area of research offer little guidance, as to which prerequisite is more appropriate. Performance measures of prerequisite courses have been studied in various disciplines (Buschena & Watts, 1999; Butler, et. al., 1994; Cadena et. al., 2003). A remarkable discussion on the effect of prerequisite courses has been found in Potolsky, et. al.(2003).

For this study, data were collected from a Mid-Western university. Statistics is a required course for all business and economics majors at this university. Statistics course stresses application of statistical concepts to decision problems facing business organizations. All sections of this course taught at the college of business use a common text book and cover the same basic topics. The course includes descriptive statistics, probability concepts, sampling processes, statistical inference, regression, and nonparametric procedures. Among the several available prerequisite courses we analyze the differential effect of Applied Calculus and Calculus-I on the Statistics course performance. Since, this will be fascinating to observe if there is any differential effect due to different arrangement of calculus course. If so, what is the propensity of the differential effect?

We hypothesize that students' performance in Statistics course as measured by the final course grade varies due to the diverse preparedness by different prerequisite courses. The question that we ask is that whether Applied Calculus or Calculus-I are availing themselves to the same background knowledge and prepare students equally for the Statistics course. Specifically, this research addresses the question; does the different mathematical background knowledge attained by students from Applied Calculus or Calculus-I create a differential effect on their performance in the Statistics course? Applied Calculus covers non-linear functions, intuitive differential, integral and multivariate calculus applications. Calculus-I covers Polynomial, exponential, logarithmic, and trigonometric functions; Differentiation with associated applications; Introduction to integration with applications.

Business and Economics students in general try to avoid (or delay) taking Statistics course. The fear of statistics may be a result of lack of acquaintance in mathematical thinking (Kellogg, 1939). Therefore, a proper prerequisite course that can build confidence against mathematical anxiety and develop mathematical thinking could help alleviate these problems. Although both prerequisite courses considered in this study are calculus, we perceive that students obtain a higher level of "mathematical maturity and thought process" from the traditional calculus than applied calculus. The reason may be traditional calculus takes students' into the journey of deeper level of quantitative reasoning compared to the applied calculus.

Authors in this study analyze the differential effect of background knowledge accumulated from two different prerequisite courses on students' performance in Statistics course. Results from the Chow-test provided strong justification for differential effect on Statistics course performance due to different mathematical background and the null hypothesis of equality of two different regression models could be rejected. This result enabled consideration to be given to traditional Calculus as a prerequisite for Statistics when advising students to accumulate background knowledge that develops quantitative reasoning skill. They found that students who took the traditional Calculus obtain higher average grades in Statistics than did students who took applied Calculus. Furthermore, their analysis reveals that students with Calculus-I background starts at an advantageous position with higher intercept value (see, Table 2B and Table 2C) compared to those with Applied Calculus. This finding implies that traditional calculus may be more effective in building quantitative concepts and reasoning.

DATA AND METHODOLOGY

Data were collected from the records of all students enrolled in the Statistics course for three consecutive semesters. Students were grouped by the prerequisite courses completed prior to enrolling in Statistics course. There were no recruitment (or selection) attempts to draw students into either of these courses. As there is no indication presented to the student about the prerequisite course, nor there is any control for which students enrolled in which course. For these reasons, it will be assumed that the students are of comparable mathematical abilities when taking a prerequisite course.

Performance comparisons are made between these two prerequisite courses (Applied Calculus and Calculus-I) on the basis of Statistics course grade. Course grades are classified in the usual manner: A, B, C, D, and F. For the purpose of comparing the average grades of the course in question, the grades assumed the standard quantitative values. An A was weighted at 4 points, a B at 3 points, a C at 2 points, a D at 1 point, and an F at 0. Students were grouped into two different groups— 1) Calculus-I and 2) Applied Calculus.

The objective of this paper is to observe the differential performance in Statistics (ST) course as a result of generating background knowledge from Applied Calculus (AC) or Calculus-I (CL). We perform Chow-test to analyze the differential effect due to different prerequisite courses. The **Chow test** (see Chow, 1960; Gujarati, 1970) is a statistical test to test the equality of regression coefficients in two different linear regression models for two different data sets. In program evaluation, the Chow-test is often used to determine whether the independent variables have different impacts due to different subgroups of the population. In our study, we examine the differential effect of two different prerequisite courses taken by two different groups of students.

The specification of the regression model for our analysis purpose can be of the following form:

$$STG_i = \alpha + \beta MATG_i + \varepsilon_i \qquad i = 1, \ldots . n \quad (1)$$

$$STG_{AC,i} = \alpha_{AC} + \beta_{AC} ACG_{AC,i} + \varepsilon_{AC,i} \qquad i = 1, \ldots . n_{AC} \quad (2)$$

$$STG_{CL,i} = \alpha_{CL} + \beta_{CL} CLG_{CL,i} + \varepsilon_{CL,i} \qquad i = 1, \ldots . n_{CL} \quad (3)$$

where, equation (2) and equation (3) are representing Applied Calculus and Calculus-I respectively and equation (1) is for both groups combined. STG denotes Statistics grade, ACG for Applied Calculus grade, CLG for Calculus-I grade, and MATG for combined mathematics (both calculus) grade.

Therefore, the null hypothesis of Chow-test asserts that both intercepts and slopes are equal, i.e., H_0 : $\alpha_{AC} = \alpha_{CL} and \beta_{AC} = \beta_{CL}$. Thus, the structure of the Chow-test takes the form:

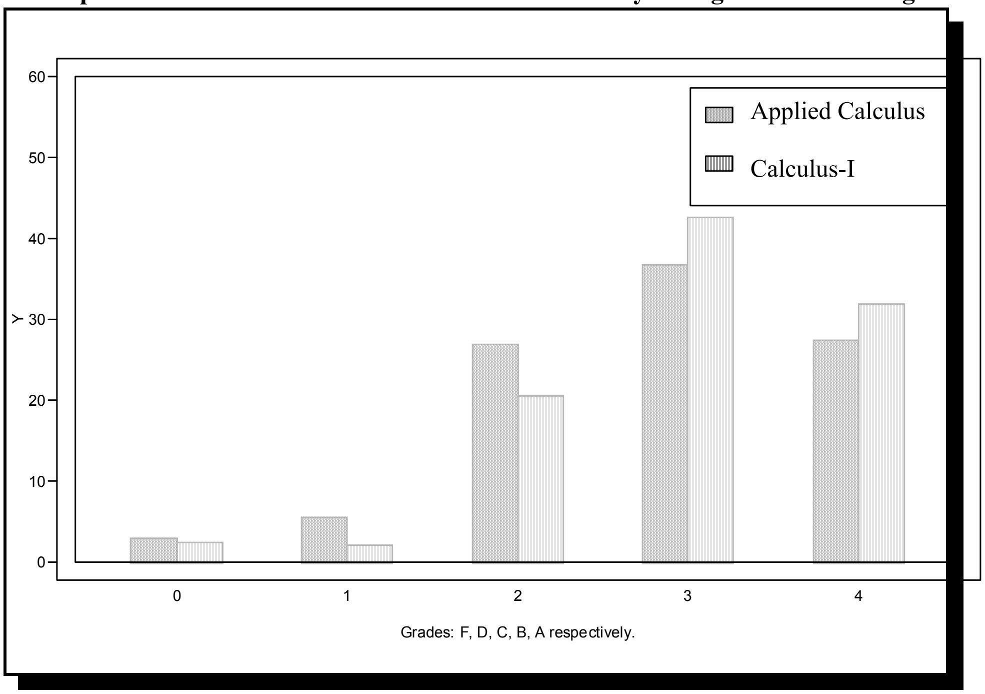
$$\frac{\left[\frac{\{S_{MAT} - (S_{AC} + S_{CL})\}}{k}\right]}{\left[\frac{(S_{AC} + S_{CL})}{(N_{AC} + N_{CL} - 2k)}\right]}$$

where, S_{MAT} be the sum of squared residuals from the combined data, S_{AC} be the sum of squares from the Applied Calculus group, and S_{CL} be the sum of squares from the Calculus-I group. N_{AC} and N_{CL} are the number of observations in each group and k is the total number of parameters (in this case, 2). This test statistic is then follows the F distribution with k and $N_{AC} + N_{CL} - 2k$ degrees of freedom.

EMPIRICAL RESULTS

We present Statistics course grade distributions in Graph 1 for both background (prerequisite) courses. The letter grade distributions in Graph 1 reveal that higher percentage of students who took Calculus-I received a better grade (A or B) in Statistics course than those who took Applied Calculus. As for example, who took Calculus-I, 74.00% received an ‘A’ or “B’ in Statistics course. In contrast, only 64% of those who took Applied Calculus received an ‘A’ or “B’ in the Statistics course. This difference reverses when we compare them for lower grades, such as C or D (see Graph 1). About 33% of Applied Calculus students received either a ‘C’ or ‘D’ in the Statistics course while only 23% of the Calculus-I students received these low grades.

Graph 1: Grade Distributions of Statistics Course by Background Knowledge.



1st bar: Statistics performance attributable to applied calculus.
2nd bar: Statistics performance attributable to calculus-I.

In Table 1, we present summary statistics for all course grades. Although, students acquired higher grade on average in Applied Calculus than Calculus-I course (2.63 vs. 2.46). We observe that there is a difference in average grade points in Statistics course between students with Applied Calculus and those with Calculus-I prerequisite. Specifically, in general students perform better in Statistics course with Calculus-I background than Applied Calculus. For example, those who took Calculus-I as a prerequisite received an average grade of 3.0 in Statistics course compared to 2.8 for those who had Applied Calculus. These results suggest that Calculus-I leads to accumulate quality human capital in terms preparedness for Statistics course and results in substantially better performance. This provided the basis to perform hypothesis test on the differential effect on Statistics course performance as a result of different calculus background. Since, the outcome of prerequisite selection has a substantial payoff, it is important for us to test the hypothesis and identify the prerequisite that has higher incremental impact on Statistics course performance.

TABLE 1: Summary Statistics by Courses

Grade	Applied Calculus Grade	Calculus-I Grade	Both Calculus Combined Grade	Statistics Grade Applied Calculus]*	Statistics Grade [Calculus-I]*	Statistics Grade [Both Combined]*
Average	2.63	2.46	2.59	2.80	3.00	2.85
Median	3.00	2.00	3.00	3.00	3.00	3.00
Std	1.00	1.07	1.02	1.00	0.92	0.99
N	659	221	880	682	237	919

Note: Maximum grade is 4 and minimum grade is 0, on a four-point scale.
* Statistics course grades with respective prerequisites; applied calculus, calculus-I and both combined.

To test the differential effect (if any) due to two different calculus backgrounds we perform the Chow-test as below. First, we run a regression for the combined (both Calculus-I and Applied Calculus) calculus background and the estimated model is:

Regression model (with both Calculus):

$$STG_i = 1.93421^* + 0.35832^* MATG_i \qquad (4)$$

* Statistically significant at better than 1% level (see Table-2A)
where, S_{MAT} = sum of squared residuals (combined) = 731.315.

Combined estimated regression model above is highly statistically significant with a positive intercept and slope. This implies if their performance is better in Calculus then the performance in Statistics course will also be superior and the rate of increase is about 1/3 of a grade point (i.e., 0.35832).

TABLE-2A: Regression Results of Statistics Course Performance Attributable to Combined (both Calculus) Background					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	117.77136	117.77136	141.39	<.0001
Error	878	731.31500	0.83293		
Corrected Total	879	849.08636			
R-Square	0.1387		Adj R-Sq	0.1377	
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	1.93421	0.08382	23.08	<.0001
MATH	1	0.35832	0.03013	11.89	<.0001

Consequently, we run both regression models separately to observe the difference in the intercept and slope due to different courses as background knowledge. If no difference exists, then we can postulate that there is no differential effect due to different calculus courses on the Statistics course performance. Estimated models are provided below:

Regression model (with Applied Calculus):

$$STG_{AC,i} = 1.74244^{*} + 0.40781^{*} ACG_{AC,i} \qquad (5)$$

* Statistically significant at better than 1% level (see Table-2B)

Where, S_{AC} = sum of squared residuals (applied calculus) = 549.376.

Regression model (with Calculus-I):

$$STG_{CL,i} = 2.37216^{*} + 0.25506^{*} CLG_{CL,i} \qquad (6)$$

* Statistically significant at better than 1% level (see Table-2C)

Where, S_{CL} = sum of squared residuals (Calculus-I) = 167.676.

TABLE-2B: Regression Results of Statistics Course Performance Attributable to Applied Calculus Background					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	110.03802	110.03802	131.59	<.0001
Error	657	549.37624	0.83619		
Corrected Total	658	659.41426			
R-Square	0.1669		Adj R-Sq	0.1656	
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	1.74244	0.10004	17.42	<.0001
MATH	1	0.40781	0.03555	11.47	<.0001

Results of these regression models have been reported in Table-2B and Table-2C. Although, both models are highly statistically significant with positive intercepts and slopes. As expected, intercept is higher with Calculus-I compared to Applied Calculus (2.37 vs. 1.74). This result is consistent with the summary statistics reported in Table 1. This implies that students with Calculus-I background starts at an advantageous position which is more than half a point (.63) higher as oppose to students with Applied Calculus background. To establish this differential effect statistically, we calculate the following test statistic to perform the Chow-test.

$$F = \frac{\left[\dfrac{\{S_{MAT} - (S_{AC} + S_{CL})\}}{k}\right]}{\left[\dfrac{(S_{AC} + S_{CL})}{(N_{AC} + N_{CL} - 2k)}\right]} = \frac{\left[\dfrac{\{731.315 - (549.376 + 167.676)\}}{2}\right]}{\left[\dfrac{(549.376 + 167.676)}{(659 + 221 - 4)}\right]} = 8.712$$

Thus, the observed test statistic F=8.712 exceeds the critical test statistic F=4.61at 1% significance level with 2 and 876 degrees of freedom. Therefore, the null hypothesis of equality of intercepts and slopes is rejected. This implies that the two regression models are different, suggesting that there is a differential effect attributable to different calculus backgrounds. These tests results lead us to conclude that students with added traditional calculus orientation do possess greater statistical proficiency. Perhaps, it is that enhanced mathematical maturity developed from the traditional calculus leading to a better understanding of statistical reasoning that resulted in elevated advantageous position for these students.

TABLE-2C: Regression Results of Statistics Course Performance Attributable to Calculus-I Background					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	16.32373	16.32373	21.32	<.0001
Error	219	167.67627	0.76565		
Corrected Total	220	184.00000			
R-Square	0.0887		Adj R-Sq	0.0846	
Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	2.37216	0.14817	16.01	<.0001
MATH	1	0.25506	0.05524	4.62	<.0001

CONCLUSION

Findings of this study suggest that prerequisite is an important component in predicting academic performance in Statistics course. Specifically, we have found that students who took the Calculus-I received higher average grades in Statistics than students who took Applied Calculus. Our analysis illustrates the importance of selecting a proper and more relevant prerequisite course for business and economics majors. This selection process of prerequisite course matters in two ways. First, the proper prerequisite course provides students with required and relevant quantitative background knowledge needed to succeed in the Statistics course(s), and consequently be beneficial for other quantitative oriented business and economics courses. Second, the prerequisite course needs to have necessary components and topics included (including the course arrangement), so that, students have better opportunity to improve their mathematical maturity needed for quantitative reasoning courses.

Therefore to improve students' performance in Statistics course, Calculus-I may be more appropriate prerequisite than Applied Calculus. Thus, it appears from our analysis that students with traditional calculus orientation may have greater statistical proficiency than with applied calculus. In addition, our analysis also reveals that students with Calculus-I background starts at an advantageous position as oppose to students with Applied Calculus background.

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