# TEACHING STICKY PRICES TO UNDERGRADUATES 

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#### Abstract

In this paper we describe a simple way of conveying to undergraduates the central message of Ball and Romer's paper "Sticky Prices as Coordination Failure." For the simple framework we use, we also find an asymmetry between adjustments to decreases versus increases in the money supply, with the former, as in Ball and Romer, creating a coordination game, while the latter gives rise, in our formulation, to a game of chicken.


## INTRODUCTION

In this paper we describe a simple way of conveying to undergraduates the central message of Ball and Romer's paper "Sticky Prices as Coordination Failure." For the simple framework we use, we also find an asymmetry between adjustments to decreases versus increases in the money supply, with the former, as in Ball and Romer, creating a coordination game, while the latter gives rise, in our formulation, to a game of chicken.

The model is simple. We have monopolistic competition with all of a large number of firms facing the same inverse demand curve of the form

$$
\frac{\pi}{3}=\frac{N}{A}-Q_{i}
$$

where $\mathrm{P}_{\mathrm{i}}$ is the $\mathrm{i}^{\text {th }}$ firm's price, P is the average price, M is the money supply, and Qi is the $i^{\text {th }}$ firm's quantity ${ }^{1}$. Firm i's sales depend negatively on its relative price and positively on the real money supply. The latter dependence can be rationalized as a real balance effect, on the basis of money in the utility function, or indirectly through the effect of real money on interest rates and interest-sensitive spending. To make things simple, we assume that firms have no variable costs, so that they maximize profits by maximizing revenue. It is then easy for students to see that the optimal relative price is $1 / 2$ the vertical intercept, which in this case is $\mathrm{M} / 2 \mathrm{P}$; while the optimal quantity is $1 / 2$ the horizontal intercept. Since we have given the inverse demand a slope of -1 , the horizontal intercept is also $\mathrm{M} / \mathrm{P}$.

Since all firms are identical, the long run relative price must be 1 , so the real money supply must be 2 as depicted in the graph below.


In the absence of menu costs, this equilibrium comes about as follows. Suppose we reduce M to $2 / 3$ its former value, shifting the demand curve down and reducing the real money supply to $(2 / 3) * 2=4 / 3$ so each would want to reduce his price to $2 / 3$ of what others are charging. But as prices fall, the real money supply increases, shifting the demand curve up again. When prices are $2 / 3$ of their former value, the real money supply is 2 again, so each will want to set a relative price of 1 , and we have our new equilibrium. In the absence of menu costs, then, money is clearly neutral. Similarly, an increase in M would increase the optimal relative price above 1 ; prices would rise in proportion to M .

Now let's add menu costs and stick with the same reduction in M.


If none adjust, th $\epsilon_{1 / 3} \quad$ firm is on the lower demand curve. Should they adjust or not i.e., is not adjusting the tuo cousonse to all others not adjusting? By lowering relative price to $2 / 3$, they would have profits of $2 / 3(2 / 3)=4 / 9$. By maintaining price, charging a relative price of 1 , they sell $1 / 3$ and get profits of $1 / 3$. Thus if menu costs exceed $1 / 9$, they do better by not adjusting, and we have an equilibrium where no one adjusts.

But now suppose that all adjust. Is this a Nash equilibrium? In this case, the typical firm is on the higher demand curve. Adjusting, which means charging a relative price of 1 given that all others have cut price by $1 / 3$, gives them profits of $(1)(1)=1$. If they do not change price, then they will have a relative price of $\mathrm{P}_{o} /(2 / 3) \mathrm{P}_{o}$ where $\mathrm{P}_{o}$ was the price all were charging before the change. With a relative price of $3 / 2$, they sell $1 / 2$, for profits of $3 / 4$. So they will want to adjust if menu costs are less than $1 / 4$ and not adjust otherwise. There is a range of menu costs, between $1 / 9$ and $1 / 4$, for which we have two symmetric Nash equlibria, one where all adjust and another where none adjust. The second equilibrium, where prices are sticky, is a coordination failure. This illustrates the main message of Ball and Romer's paper in a simple framework that undergraduates can grasp.

The key to the result is that the benefits of adjusting, gross of menu costs, are greater when demand is higher. Demand is higher when all adjust then when none do. Benefits of adjusting net of menu costs may thus be negative when none adjust and positive when all adjust.

In our model, the case of an increase in the money supply is different. Suppose we have the money supply increase to $3 / 2$ times its initial value. The graph below illustrates.


If all adjust, we are on the lower demand curve. The typical firm would have profits of 1 if it adjusts. If it does not adjust, it will have a relative price of $\mathrm{P}_{\mathrm{o}} /(3 / 2) \mathrm{P}_{\mathrm{o}}=2 / 3$ and can sell $4 / 3$ units for profits of $8 / 9$. Thus it would not want to adjust if menu costs exceeded $1 / 9$. In other words, if all adjust and menu costs exceed $1 / 9$, it is not a best response to adjust. All adjusting is not a Nash equilibrium, therefore.

If none adjust, each firm would face the higher demand curve. Adjusting optimally requires a relative price of $3 / 2$ for profits of $3 / 2(3 / 2)=9 / 4$. Failing to adjust means setting a relative price of 1 and selling 2 units, for profits of 2 . Thus if menu costs are less than $1 / 4$, each wants to adjust when other do not adjust; there is no symmetric equilibrium in which none adjust.

Instead of a coordination game, in this case we have a game of chicken. In the appendix (We generally teach this without the appendix material, although the appendix requires nothing particularly advanced.), we show that the unique symmetric Nash equilibrium has each adjusting with probability equal to $(1 / \mathrm{MC})^{1 / 2}-2$, where MC is the menu cost. Alternatively, the equilibrium has this proportion of firms adjusting, while the others do not.

It is easy to see why the two cases, decreases versus increases in the money supply, are so different. In the case of an increase in M , demand is greater, and thus the costs of sub-optimal adjustment are greater, when none adjust. For a decrease in $M$, the costs of sub-optimal adjustment are greater when all adjust.

It is interesting, too, that in this simple context, we get a micro foundation for the old Keynesian idea that the aggregate supply curve is steeper for increases in output than for decreases (The "L-shaped" AS function beloved of many old Keynesian texts.). Hence decreases in the money supply move us along a horizontal AS if we get the sticky price equilibrium, while increases in M move us along an upward sloping, though not vertical, AS curve, since prices adjust but only partially.

## END NOTE

1. One of the desirable features of Ball and Romer is that they have started from utility maximization to obtain the demand curves for their consumers. One might wonder if demand functions such as what we have could be the outcome of utility maximization. It turns out that in an $n$ good world, $n-1$ demands can have the form $\mathrm{Q}_{1}=\left(\mathrm{M}-\mathrm{b}_{\mathrm{i}} \mathrm{P}_{\mathrm{i}}\right) / \mathrm{P}_{\mathrm{n}}$ and arise from utility maximization. While these demand functions are not exactly the ones we have above, they suggest that the results we get are consistent with standard assumptions. Details of this computation can be obtained from the authors upon request.

## REFERENCES

Ball, Laurence and David Romer, "Sticky Prices as Coordination Failure," American Economic Review, 81(3) 539552.

## APPENDIX

## Case 1: M decreases to $2 / 3$ of its old value

Suppose $\lambda$ proportion of firms reduce money price by $1 / 3$. First we show that this will give them the optimal relative price. We start from $\frac{N_{0}}{D_{0}}=2$. We now have $\mathrm{M}_{1}=2 \mathrm{M}_{0} / 3$.

$$
\text { So } \frac{1 M_{6}}{2 P_{2}}=\left(\frac{1}{2}\right)\left(\frac{2}{(8-2)}\right) z=\left(\frac{2}{8-2}\right)=\text { optimal }^{\frac{P_{i}}{P}}
$$

Since the adjusters have $\mathrm{P}_{\mathrm{i}}=\frac{\frac{\partial}{2}}{\frac{2}{2}} P_{Q_{0}}$ and $\mathrm{P}_{1}=\left(1-\frac{\lambda}{3}\right) \mathrm{P}_{0}$, the adjusters have relative price:

$$
\frac{B_{1}}{B_{6}}=\frac{2_{g} B_{0}}{1-\frac{d}{2} \bar{F}_{i}}=\left(\frac{2}{3-\lambda}\right), \text { which is optimal as shown above. }
$$

Now we want to find the payoff to adjusting and the payoff to not adjusting both as functions of $\lambda$, the proportion who adjust. Here is the picture.


$$
\begin{aligned}
& \mathrm{P}_{1}=\left(\lambda\left(\frac{2}{8}\right)+(1-\lambda)(1)\right) F_{\varrho}\left(1-\frac{2}{8}\right) F_{\varnothing}
\end{aligned}
$$

As noted above, when $\lambda$ proportion adjust, the new real money supply $=\left(\frac{2}{(2-2)}\right) \frac{M_{8}}{E_{0}}=\left(\frac{2}{(2-22}\right) 2=$ $\left(\frac{4}{(2-22)}\right)$

The adjusters set relative prices at $\left(\frac{2}{(3-2)}\right)$ and sell $\left(\frac{2}{(3-2)}\right)$ units, earning $\left(\frac{1}{(\Omega-2)^{2}}\right)$ - mc where mc is the menu cost.

The non-adjusters will have relative price $=\frac{F_{8}}{F_{4}}=\frac{B_{0}}{1-\frac{4}{p_{0}}}=\frac{1}{1-\frac{4}{7}}=\frac{2}{2-2}$. They sell $\frac{1}{3-2}$ units and pay no menu costs, earning $\left(\frac{y}{8-22^{2}}\right)$. We have two payoff functions, one for adjusters, $\pi=\left(\frac{\pi}{(2-2)^{2}}\right)$ - MC, and one for non-adjusters, $\pi=\left(\frac{\Omega}{(3-2)^{2}}\right)$

Graphing both payoff functions gives us the following.


The payoff functions intersect at $\lambda^{*}=\frac{3-\left(\frac{1}{m s}\right.}{)^{\frac{6}{2}}}$ under the assumption that ${ }^{\frac{1}{7}}<\mathrm{mc}<\frac{1}{4}$. If $\mathrm{mc}<1 / 9$, then the adjuster's profit function lies everywhere above the non-adjuster's, so the only equilibrium has all adjusting. If mc exceeds $1 / 4$, on the other hand, then the situation is reversed - the non-adjuster's profit is then everywhere above the adjuster's, so $\lambda=0$ is the only equilibrium - none will adjust. With our assumptions, both $\lambda=$ 0 and $\lambda=1$ are equilibria, since at $\lambda=0$, not adjusting give greater profit than adjusting, while at $\lambda=1$, adjusting give the greater profit.

There is also a mixed strategy equilibrium here which has each firm adjusting with probability $\lambda^{*}$. This is because when other firms will adjust with probability $\lambda^{*}$, the proportion adjusting (among a large number of firms) will be $\lambda^{*}$, so that adjusting with probability $\lambda^{*}$ is a (weak) best response given the firm's indifference between adjusting and not at $\lambda^{*}$. But $\lambda^{*}$ is clearly unstable. If $\lambda>\lambda^{*}$, even slightly, the payoff to adjusting exceeds the
payoff to non-adjusting, so all will adjust. If $\lambda<\lambda^{*}$, even slightly, the payoff to not adjusting exceeds the payoff to adjusting so none will adjust. The two stable equilibria are $\lambda=0$ and $\lambda=1$.

## Case 2: $M$ increases to $\mathbf{3} / \mathbf{2}$ its former value

Here we have $M_{1}=(3 / 2) M_{0}$. Again let $\lambda$ proportion increase money price to $3 / 2$ its initial value. The new price level, $\mathrm{P}_{1}$ is

$$
\begin{aligned}
& \mathrm{P}_{1}=\lambda(3 / 2) \mathrm{P}_{\mathrm{o}}+(1-\lambda) \mathrm{P}_{\mathrm{o}}=\left(1+\frac{\lambda}{2}\right) \boldsymbol{P}_{\odot} .
\end{aligned}
$$

The optimal relative price is, then, $\frac{\vdots}{2} \frac{M}{V}$, as usual, or $\left(\frac{2}{2+2}\right)$. Note that those who have increased money $\frac{\frac{x^{2}}{2} p_{0}}{\left(1+\frac{4}{2}\right) p_{0}}=\left(\frac{y}{2+2}\right)$ and so have indeed adjusted optimally.
prices as supposed have relative prices of

$$
\frac{B_{0}}{\left(1-\frac{1}{2}-B_{x}\right.}=\left(\frac{2}{2+2}\right) .
$$

Here is the demand curve for the typical firm:


Thus the payoff to adjusting is: $\left(\frac{2}{2+2}\right)\left(\frac{2}{2+2}\right)-m c=\frac{9}{(2+2)^{2}}-m c$.

The payoff to not adjusting is: $\left(\frac{2}{2+5}\right)\left(\frac{4}{2+\pi}\right)=\frac{\beta}{(2+2)^{2}}$. Again, assuming $1 / 9<\mathrm{mc}<1 / 4$, the payoff functions can be drawn as shown below.


Here neither 0 nor 1 are equilibria since at 0 , the payoff to adjusting exceeds the payoff to not adjusting, while at 1 , the reverse is true. We now have a stable equilibrium at $\lambda^{*}$, since for $\lambda>\lambda^{*}$, non- adjustment has a higher payoff, so $\lambda$ falls; while for $\lambda<\lambda^{*}$, adjustment has a higher payoff, so $\lambda$ increases. It is easy to see that the discussion in the main text of the paper follows this appendix for the special cases of $\lambda=0$ and $\lambda=1$.

