STUDENT CHOICE OF EFFORT IN PRINCIPLES OF MACROECONOMICS

Steven Dickey, Eastern Kentucky University
Robert G. Houston Jr., Eastern Kentucky University

ABSTRACT

The model of student effort choice implicit in the literature is a tradeoff between the utility of scoring well on examinations and the disutility of the effort expended studying. The existing literature contains only minimally specified model structures. This paper develops the implicit model in the literature with an explicit utility maximization problem. The solution to the student's choice of effort is then empirically estimated with a unique and much broader data set. The results provide a more complete perspective on the factors determining student choice of effort. The model is then extended with estimation of a production function allowing for a calculation of the marginal effects that each of the variables ultimately has on score through its impact on effort.

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INTRODUCTION AND LITERATURE REVIEW

Student effort is recognized as an important input in education production function. Although effort is essential in the theoretical modeling of education production, the direct treatment of effort has been limited both theoretically and empirically. Student effort has been modeled in the literature by McKenzie and Staaf [1974], Wetzel [1977], Becker [1982], Becker and Rosen [1992] and Krohn and O'Connor [2005] . A limitation found in these previous studies are they are focused principally on modeling educational production and give effort limited attention and tend not to empirically estimate effort at all.

Empirical estimations are limited to studies by Wetzel [1977] and Krohn and O'Connor [2005]. These empirical studies offer limited explanations of the connection of the regressors to the utility function of the student or to the educational production function. Wetzel [1977] estimates regressions where the dependent variables are indirect measures of effort. Wetzel constructs three McKenzie and Staaf [1974] styled effort variables by dividing gain in TUCE scores by three different aptitude scores based on the SAT as a proxy for effort. Wetzel uses end of semester TUCE score and never directly observes student effort. In the Wetzel study, explanatory variables are limited to student grade expectation and hours worked as predictors of student effort. Wetzel finds student work hours has a negative impact on effort and grade expectation has a positive impact on effort. More recently, Krohn and O'Connor [2005] estimate student effort with an actual observation of effort rather than a McKenzie-Staaf proxy. However, the independent variables used to estimate effort are limited to a small vector of human capital measures, GPA, SAT and previous classes in economics. Other regressors include the pretest score and a dummy variable for gender. Krohn and O'Connor find students with higher ability study more. They also find evidence that females may in fact put forth more effort and that higher exam scores earlier in the semester lead to less effort exerted later in the semester. Overall, the collective right hand specification in this literature is thin and the development of this topic has been limited.

The purpose of this paper is to add to the existing literature by developing a more thorough specification of the model structure and provide an explicit connection of the empirical estimation of effort to the underlying student utility function. The resulting model provides a more complete perspective on the vectors determining student choice of effort. The empirical model will then be estimated using a richer list of explanatory variables than has previously appeared in the literature. The results will be used to calculate both marginal effects on post test scoring and the actual learning differentials implicitly observed in our data. This approach provides a fuller presentation of the determinants of effort in both theoretical utility maximization and the observed impact of effort determinants on learning.

MODEL

The model of student choice implicit in the literature is a tradeoff between the utility of the student's post test score ($S1$) and the disutility of the student's effort (E), the student's utility/disutility tradeoff. While this literature poses the problem as a utility/disutility tradeoff, the disutility of effort is a surrogate for the opportunity cost of effort in addition to any unpleasant aspect of the work itself. The disutility of effort is net of any pleasant aspect to the work itself. A student's post test score depends on the student's pretest score ($S0$), the rate of depreciation (d) of pretest understanding and the student's gain (G) from effort. Equation 1 shows this relationship.

$$1.\ S1 = G + (1-d)S0$$

While we note the rate d above, we have no reliable measure of how knowledge prior to day one of the class depreciates across students. This rate is either implicitly or explicitly assumed constant in all studies in this literature. We will follow this practice here. Therefore, we are treating this rate as an unobserved parameter of the student's maximization problem.

Variables influencing a student's marginal values (preferences) in the tradeoff are represented with the vector of variables *P*. Equation 2 summarizes the utility function *U*.

$$2.\ U(G + (1\text{-}d)S0, E, P)$$

A student's expected production depends upon teacher inputs, *T*, human capital, *K*, student effort, *E*, and the student's perception of the relative difficulty of the material, *PRD*. The student's expected gain function (*G*) is shown in equation 3 below. A student maximizes welfare under the condition shown in equation 4 involving the marginal utility of gain, U_G, the marginal gain from effort, G_E, and the marginal disutility of effort, U_E. The solution to the student's choice is given by the effort function shown in equation 5.

$$3.\ G(T, K, E, PRD)$$
$$4.\ U_G(S0, P)\ G_E(T, K, E, PRD) - U_E(E, P) = 0$$
$$5.\ E = f(S0, PRD, T, K, P)$$

The expected signs of the variables in the effort function, equation 5, can be determined by inspection of equation 4[1]. A high value of *S0* means that the student is further into the region of diminishing marginal utility of posttest score and would imply a negative coefficient for *S0*. Teaching inputs may be complimentary to student effort (Increasing *T* increases G_E.) implying a positive coefficient on the *T* regressor. Alternatively, teaching inputs may be substitutes for student effort (Decreasing *T* decreases G_E.) implying a negative coefficient on the *T* regressor. Human capital *K* always increases G_E and so implies a positive coefficient on *K* regressors. Higher values for *PRD* mean a reduced reward to student effort and imply a negative coefficient on that regressor. In theory PRD is an ex-ante concept, however, our survey measures it ex-post. This may blur the lines of cause and effect. None the less, we feel that it is an important influence on the student's choice. Therefore we will assume an ex-ante character in student responses. Sign expectations for the coefficients of P regressors depend on whether the regressor would be expected to shift the utility/disutility tradeoff toward or away from effort. This formulation of the model allows us to use the vector list inside the function *f* as an expositional scheme in specifying regressor variables. Each vector, *T*, *K* and *P,* will be discussed in turn. We shall also specify an additional vector (*N*) of three regressors that indicate both productivity and preferences.

The vector T consists of variables representing teaching influences. Two teaching variables were based on student perceived clarity in the reading of the textbook, *CREAD*, and clarity in the lecture, *CLEC*. Our third teaching variable was the student's assessment of the rapport between the teacher and the students, *RAP*. These variables are measured with Lichert scale survey questions. Lastly, a dummy variable for the instructor (TEACHER) was included.

As noted earlier, the signs of these regressors depend on the input being a compliment or a substitute for student effort.

The vector K consists of measures of human capital. Human capital accumulated prior to entering university study is measured by the student's *ACT* composite score. ACT enters our regressions in log form. Accumulated hours of course work, *AHRS*, is used to represent experience with college courses. *AGE* is used to represent maturity. Another way of employing age is to distinguish between traditional students (age < 25) and nontraditional students with the binary variable *NONTRAD*.

The vector P (preference for achievement over leisure) involves family influences and income (included as a separate regressor). Family influences (father/mother etc.) occur in complex ways. When these influences enter the classroom it is only in student preferences for achievement over leisure. In absence of family measures, we can do well in representing such preferences with the ratio GPA/ACT. While our model is about the student's choice of effort for an individual class, we can apply the same reasoning to a model of a student's overall GPA. Suppose a simple linear utility function, a Cobb-Douglass style production function and the solution equation for GPA/ACT in equations 6 through 8 below.

6. $U = a_1 GPA + a_2 E$
7. $GPA = BE^{b} ACT^{1-b}$
8. $GPA/ACT = B(Bb)^{b/(1-b)}[a_1/a_2]^{b/(1-b)}$

In the utility function, a_1/a_2 is the student's preference for achievement over leisure. Within the limitations of this simple construction, GPA/ACT is a monotonic transformation of the student's preference for achievement over leisure. When family measures are not available we will use GPA/ACT and where they are available we will use these measures along with GPA/ACT.

Peer influences are represented by the percent of the class with the same major, *SMAJ*. A Lichert scale measure of student preference for working in short periods of intense effort, work style preference (*WP*) is also included in measuring a student's preferences in the utility/disutility tradeoff.

The vector N includes regressors that are nonspecific indicators of both productivity and preferences in the tradeoff. Male gender, *MALE*, is one such binary variable. Having a high percentage of accumulated hours transferred from other universities, *TRAN*, is another. Course specific motivation and ability is measured by a binary for being a non-business major, *NONBUS*.

DATA

The model presented in the previous section was empirically tested using student examination and survey data collected in macroeconomic principles classes at a public university in Kentucky. Students were given a pre-test covering the basics of aggregate demand, aggregate supply, short-run equilibrium and long-run self-adjustment[2]. The test was given on the first day of class and not returned to students. The test consisted of thirty-five multiple-choice questions selected by topic from the textbook test bank. Reading was assigned and then material was presented in traditional lecture format. Students were surveyed each class period on the time they spent studying the material since the previous class meeting (i.e. reading, going over notes working problems, etc.). After the material was covered, the test was re-administered to students about six weeks into the semester, students were not aware that the same test was to be given. The two instructors spent the same amount of class time covering material and utilized the same textbook.

A distinction between this study and other studies in the literature is in the measurement of effort. Studies prior to ours are often end-to-end in nature and measure learning by the overall gain in understanding at the end of the course and thus ask students' questions like, "How much did you study on average per week for this class?" This study directly surveys student study minutes through self-reporting. This shorter periodicity of data collection (i.e. at the start of each class period) offers potential gains in measurement accuracy because the reported events are more proximate. The experimental design prevented incentive for over or under reporting of study minutes. Both of our data collection included this effort measurement.

Clearly the measure of student study minutes does not reflect the quality dimension of effort. On the other hand, one cannot imagine a rational motivation for students to spent time pointlessly. If students fail to make their efforts effective, i.e as a pure cost with no benefit, it would indicate an irrational choice. Empirically this type of variance will be assigned to the error term of our regressions.

In addition to the student survey, information on *GPA*, *ACT* scores, and *AHRS* were collected from the university. Some data points were lost due to the failure of the student to take one of the exams, failure to submit effort data or unavailable transcript information for the student. The data described above are the consequence of our previous inquiries into education production in the principles class. At the midpoint of collecting production data, we decided to expand our inquiry to include student effort functions. In particular, family data and student perceptions of teaching were added to the variables being collected. Therefore, our data set includes a group of observations in which we do not have the additional survey information and another group of observations where the additional survey information was collected. We refer to the former as the long data set and the latter as the wide data set. Variable definitions, data availability and summary statistics are reported in Table 1.

Table 1: Variable, Notation And Summary Statistics		
Vectors/ Variables	Description	Mean (Std. Dev.)
EFFORT	Total Minutes Study Time[a]	298.77(182.19)
S0	Pretest Score[a]	.43 (.12)
PRD	Perceived Relative Difficulty[a]	278.88 (74.81)
T	Measures of Teaching Influences	
RAP	Teacher Rapport with the Class[b]	3.65 (.76)
CLEC	Clarity of the Lectures[b]	3.91 (.91)
CREAD	Clarity of the Reading in the Textbook[b]	3.69 (.86)
TEACHER	Teacher Binary[a]	.52 (.50)
K	Measures of Human Capital	
ACT	The Student's Composite Act Score[a]	21.3 (3.55)
GPA/ACT	GPA to Act Ratio[a]	.14 (.03)
AHRS	Accumulated College Coursework[a]	65.58(32.80)
AGE	Age[a]	21.90 (2.74)
NONTRAD	A Nontraditional Student[a]	.09 (.29)
P	Variables Affecting the Student's Preferences for Utility of Scoring and Disutility of Effort	
FED	Father's Education[b]	14.03 (2.98)
MED	Mother's Education[b]	13.78 (2.91)
SIB	Number of Siblings[b]	2.26 (1.73)
SED	Number of Siblings with College Hours[b]	.77 (1.02)
INC	Family Income[b]	1.34 (1.81)
SMAJ	% Of The Class with the Same Major[a]	.10 (.07)
WP	Preference for Short Intense Periods of Effort[a]	2.19 (1.21)
N	Nonspecific Indicators of Preference and/or Productivity	
MALE	Male[a]	.61 (.48)
TRAN	% of Transfer Hours in Accumulated Hours[a]	.39 (.49)
NONBUS	A Non-Business Major Binary[a]	.37 (.48)
[A]available In Both Datasets [B]available In The Wide Dataset		

EMPIRICAL ESTIMATION

The estimation of the student effort equation using the long data set is reported in columns one and two of Table 2. The estimation of the student effort equation using the wide data set is reported in columns one and two of Table 3. In both tables, moving from column 2 to 3 we remove insignificant variables to gage the impact on remaining coefficients. In order to interpret the model further, we report the marginal effects of each variable and the corresponding maximum learning differential in our data set for each variable. This information appears in the last two columns of both Table 2 and Table 3. The marginal effect of each regressor is its coefficient in the estimation of the effort equation multiplied by the marginal product of effort derived from the production function shown below. Each regressor's range of variation multiplied by its marginal effect tells us the largest learning differential that the regressor implicitly created in our data set.

Table 2: Student Effort Equation Estimates and Impact – Long Dataset

VARIABLE/(VECTOR)	Coefficient (t-Stat)	Coefficient (t-Stat)	Marginal Effect	Learning Differential
S0	-1.77 (-4.22)	-1.67 (-4.08)	-.064	-.038
PRD	-.000 (-.10)	-.000 (-.45)	-.00001	-.006
TEACHER(T)	-.222 (-2.30)	-.241 (-2.55)	-.009	-.009
ACT(K)	.750 (2.25)	.668 (2.05)	.025	.026
AHRS(K)	.001 (.79)			
AGE(K)	.027 (.75)			
GPA/ACT(P)	4.23 (2.10)	3.71 (1.90)	.141	.026
NONTRAD(P)	.298 (.91)	.537 (3.05)	.02	.02
SMAJ(P)	-4.66 (-5.19)	-4.68 (-5.25)	-.178	-.061
WP(P)	-.040 (-1.07)	-.033 (-.92)	-.001	-.005
MALE(N)	-.170 (-1.93)	-.145 (-1.71)	-.006	-.006
TRAN(N)	.022 (.23)			
NONBUS(N)	-.294 (-2.57)	-.275 (-2.46)	-.011	-.011
R^2/n	.43/113	.41/113		

Table 3: Student Effort Equation Estimates and Impact – Wide Dataset

VARIABLE (VECTOR)	Coefficient (t-Stat)	Coefficient (t-Stat)	Marginal Effect	Learning Differential
S0	-2.28 (-4.07)	-2.43 (-4.46)	-.092	-.055
PRD	-.002 (-2.40)	-.002 (-2.12)	-.00008	-.039
RAP(T)	.006 (.06)			
CLEC(T)	-.091 (-.95)			
CREAD(T)	.229 (2.61)	.159 (2.37)	.006	.018
TEACHER(T)	-.475 (-2.59)	-.541 (-4.20)	-.021	-.021
ACT(K)	1.17 (2.25)	1.49 (3.36)	.057	.057
AHRS(K)	.002 (.48)			
AGE(K)	-.017 (-.26)			
GPA/ACT(P)	2.47 (.88)	3.06 (1.21)	.116	.021
NONTRAD(P)	.902 (1.58)	.654 (2.85)	.025	.025
FED(P)	-.033 (-1.26)			
MED(P)	.001 (.04)			
SIB(P)	-.051 (-.93)			
SED(P)	.151 (2.43)	.115 (2.35)	.004	.026
INC(P)	.068 (.99)			

Table 3: Student Effort Equation Estimates and Impact – Wide Dataset				
VARIABLE (VECTOR)	Coefficient (t-Stat)	Coefficient (t-Stat)	Marginal Effect	Learning Differential
SMAJ(P)	-4.40 (-3.56)	-5.02 (-3.75)	-.191	-.066
WP(P)	-.161 (-2.78)	-.166 (-3.14)	-.006	-.025
MALE(N)	-.275 (-1.18)	-.023 (-.19)	-.0009	-.0009
TRAN(N)	.090 (.67)			-.014
NONBUS(N)	-.177 (-.92)	-.357 (-2.17)	-.014	-.014
R^2/n	.74/58	.69/59		

Let the production function used in these calculations is given in equation 9.

$$9.\quad S1= a0+a1S0+a2Effort +a3ACT+a4Teacher$$

The estimated coefficients using the relevant data obtained from the long data set (with t-ratios in parenthesis) are shown as equation 10.

$$\begin{array}{llllll} 10.\ S1= & -.368 & +.491S0 & +.038Effort & +.210ACT & +.019Teacher \\ & (-1.54) & (5.07) & (2.05) & (2.98) & (.84) \end{array}$$

$$R2=.317 \qquad n=131$$

Based on these estimates, the marginal product of effort is positive and significant as we would expect.

A statistical issue arises from the endogeneity of effort in equation 10. Ordinarily this results in stochastic regressor bias because effort is correlated with the disturbance of the regression. In this literature, the operating assumption is that students maximize their expected utility. This literally strips the stochastic term from equation 10 in the students' choice making that choice independent of the error term. Further discussion of this point is provided in Appendix B.

The model performed well, explaining around about forty percent of the variation in the dependent variable using in the long data set and seventy percent in the wide data set. The results were stable across the two sets of estimation and were tested for heteroscedasticity using the Ramsey test and found homoscedastistic. The independent variables also performed well individually and as predicted by theory.

The coefficient on the pre-test variable, *S0*, is negative and significant at the one percent level in all of the estimates. As predicted by the comparative statics, a student achieving a high pre-test score will exert less effort in preparing for the post-test. This variable also has the second largest learning differential in both data sets. The variable measuring student perceived difficulty, *PRD*, also performed as predicted by the model. *PRD* is negative in every equation estimated. However, *PRD* is only significant in those equations utilizing the wide data set.

Two of the classroom inputs were found to significantly affect effort. The variable measuring the student perceived clarity of the reading assignments, *CREAD*, was positive and significant at the one percent level. From a student perspective a high CREAD value enhances student effort and is not seen as a substitute for it. The dummy variable capturing differences between the two instructors was significant in all the estimated equations. One of the instructors had students that exerted significantly less effort (avg. 256 minutes) compared to the other instructor (avg. 356 minutes). This instructor's presentation of the material was seen by students as a substitute for student effort.

In the vector of human capital measures, *ACT*, GPA/ACT and *NONTRAD* all had positive coefficients as predicted by the model. ACT was significant in all of the equations estimated including both the wide and long data set. GPA/ACT was only found to be significant in the long data set. GPA/ACT had the largest positive impact on learning differential in both data sets. NONTRAD was significant in the refined estimates of both the wide and long data sets. *AGE* and *AHRS* performed poorly in all of the models in which they were included. *NONTRAD*, *AGE*, and *AHRS* are likely redundant measures of experience and maturity; as a result, the two later measures were eventually dropped in favor of *NONTRAD*.

The vector of variables measuring student utility of test score on the post-test and the disutility of effort achieved mixed results. The variables *INC*, *FED*, *MED* and SIB were insignificant in all of the models estimated and eventually led to their omission in the final estimations of the model. The variables *SED*, *SMAJ* and *WP* were the significant variables from the preference vector. SED was positive and significant at the one percent level, suggesting that if students have had siblings with some college experience the more effort they will exert in preparing for the post-test. SMAJ was positive and significant at the one percent level. This suggests that the more students having the same major as a given student in the classroom will negatively influence the amount of effort exerted by the student. It is interesting that the quasi-peer influences, *SED* and *SMAJ*, are more impactful on student effort than the parental influences as measured by *INC*, *MED* and *FED*. Student work style preference, *WP*, was negative and significant at the one percent level in the wide data set only. Students, who indicate that they prefer to study intensely for short periods, actually exert less effort in total.

MALE was negative and significant in the long data set only. The impact of MALE on learning differentials, however, is small relative to other significant variables. Differences in gender achievement in economics may be an area for future research. *NONBUS* was negative and significant in the final equations estimated for both the wide and long data sets suggesting non business students exert less effort all else equal which might be expected. However, this variable has a relatively small impact on learning differentials.

CONCLUSION

This paper has added to the existing literature on student effort by expanding the specification of the effort function to include all of its principle vectors of influence. These vectors included teaching inputs (*T),* human capital (*K),* and student preferences in the tradeoff between marginal gain from effort and the marginal disutility of effort (*P).* These vectors of influence are added to the additional factors of pretest score (S0) and perceived relative difficulty (PRD). The theoretical model was improved by explicitly connecting regressors to the utility function or the production function. It was estimated using a richer list of explanatory variables than had been used in previous studies in the literature. The results are reported and used to calculate both marginal effects on post test scoring and implicit learning differentials observed in our data. This approach provided a fuller perspective on the effort function in theoretical utility maximization and in the observed impact of effort determinants on learning in our sample.

Two data structures were employed for estimation purposes. Although the wide data set is limited in observations, the overall results are strong and generally consistent with the results found in long data set. The wide data set also provides a relatively strong indication that the added variables substantially improve the model's empirical performance. The most significant negative influences on effort included: Pre-test score, student perceived difficulty, proportion of peers with the same major, preference for short intense study, and a non-business major. The most significant positive influences on effort included: clarity of the reading assignments, ACT score, number of siblings with college experience and non-traditional student status. A possible application of this model for further research could be as a vehicle for testing differences in teaching regimes or course designs in order to determine which elicits greater student effort and ultimately improved educational outcomes.

ENDNOTES

1 Comparative statistics are provided in the appendix.

2 This material is covered in Chapter 7 and Chapter 8 of Roger Arnold's *Macroeconomics*, 5th edition.

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APPENDIX A

This appendix contains the comparative static analysis referred to in the paper. Our purpose is to derive the expected signs of regression coefficients of the effort function derived from utility maximization by the student. Utility depends on S1, the post test score, and the time spent in effort, E. Maximization is constrained by the relationship between production and effort in the production function. Production also depends on the student's pretest score, S0, their human capital, K, and the student's perception of the difficulty, PRD. The problem is stated:

1. Maximize U(S1, E) subject to S1=f(PRD, K, E) + rS0

The assumption that the student's rate of retention of achievement on the pretest (1-d) is uniform across the data set reflects the lack of data on this rate. We substitute the constrained value for S1 (which is f(PRD, K, E) + (1-d)S0 for S1 in the utility function and differentiate with respect to E. In our notation, the derivative of a function k with respect to x is denoted k_x and the derivative of k_x with respect to z is denoted $k_{x,z}$. The first order condition for maximization is:

2. $dU/dE = U_{S1}{*}f_E + U_E = 0$

This condition is further differentiated by E and one of the variables to be analyzed (X is PRD, K or S0.) and then solved for the dE/dX. The implicit effort function underlying our regressions is E = g(PRD, K, S0). For each variable in g, the analyses derive its expected regression sign.

For PRD, we differentiate equation 2 by PRD and E producing equations 3 and 3'.

3. $U_{S1}*f_{E,E}*d_E + U_{S1}*f_{E,PRD}*d_{PRD} + U_{E,E}*d_E = 0$

3'. $d_E/d_{PRD} = [-U_{S1}*f_{E,PRD}]/[U_{S1}*f_{E,E} + U_{E,E}] < 0$
(-) (-) (-) (+) (-) (≤0)

The outcome of a negative expected sign corresponds to the assumptions that the marginal utility of achievement, U_{S1}, is positive, the effect of higher PRD on the marginal product of effort, $f_{E,PRD}$, is negative, the effect of more effort on the marginal product of effort, $f_{E,E}$, is negative and that the effect of more effort on the disutility of effort is no effect or a negative change.

For K, we differentiate equation 2 by K and E producing equations 4 and 4'.

4. $U_{S1}*f_{E,E}*d_E + U_{S1}*f_{E,K}*d_K + U_{E,E}*d_E = 0$

4'. $d_E/d_K = [-U_{S1}*f_{E,K}]/[U_{S1}*f_{E,E} + U_{E,E}] > 0$
(+) (-) (+) (+) (-) (≤0)

The outcome of a positive expected sign corresponds to the assumptions that the marginal utility of achievement, U_{S1}, is positive, the effect of higher K on the marginal product of effort, $f_{E,K}$, is positive, the effect of more effort on the marginal product of effort, $f_{E,E}$, is negative and that the effect of more effort on the disutility of effort is no effect or a negative change.

For S0, we differentiate equation 2 by S0 and E producing equations 5 and 5'.

5. $U_{S1}*f_{E,E}*d_E + U_{S1,S1}*r*d_{S0} + U_{E,E}*d_E = 0$

5'. $d_E/d_{S0} = [-U_{S1,S1}*r]/[U_{S1}*f_{E,E} + U_{E,E}] < 0$
-(-) (+) (+) (-) (≤0)

The outcome of a negative expected sign corresponds to the assumptions that the effect of greater achievement on the marginal utility of achievement, $U_{S1,S1}$, is negative, the marginal utility of achievement, U_{S1}, is positive, the effect of more effort on the marginal product of effort, $f_{E,E}$, is negative and that the effect of more effort on the disutility of effort is no effect or a negative change.

APPENDIX B

Our model can be expressed:
Maximize E[U(S1, Effort)] subject to :
S1= (1-d)S0 + f(Effort, ACT, Z) + v,

where v is the stochastic element of the production function. In a production function regression, v would be the error term of the regression. The E operator and brackets symbolize the student maximizing the expected value of their utility. Substituting the S1 equation into the problem and setting v to zero in order to be maximizing expected utility the student maximizes:

U((1-d)S0 + f(Effort, ACT, Z) + 0, Effort)

The resulting effort level depends on S0, ACT, and Z, but is independent of v; the covariance of Effort and v is zero. In the two regression model shown below, Effort in equation one is independent of the disturbance of that regression, v.

S1 = (1-d)S0 + f(Effort, ACT, Z) + v
Effort = f(S0, ACT, Z)

The only reason for two stage least squares is that one cov(Effort, v) is nonzero causing stochastic regressor bias under OLS. Here, it is not that way and OLS estimation does not produce biased estimates.