Managing variance in the integrated surgery and physician scheduling problem

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Abstract

Uncertainty is an element that cannot be underestimated nor ignored in health care optimiza-tion. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters. The Integrated Physician and Surgery Scheduling Problem (IPSSP) can be formulated as a stochastic programming model since the surgery duration comes with a prede ned level of uncertainty. The stochastic IPSSP is formulated as a two stage stochastic program with integer recourse, which is solved by the L-shaped method. In this paper, we evaluate the computational enhancements of the exact L-shaped method. In order to solve stochastic problems concerning many scenarios, we developed a sampling method to generate approximate solutions to the stochastic IPSSP. Simulation results prove our approach to be better than the expected value equivalent while only requiring a little more computation time, without taking into account the predictive error in the duration of the cases.

Keywords: Stochastic programming, Mixed integer linear programming, Operational research, Operations planning, Scheduling.

Introduction

In recent years, there have been a growing number of research studies aimed towards operating theatre planning. This is due both to the high costs of surgical facilities as well as the impact of their activities on the demand for the hospital services and waiting time. Planning surgical activities (pre-, peri- and post-operative) is not an easy task due to the large number of decision variables and uncertainty. Uncertainties in the OR environment arise from emergency patient arrivals, variable surgery duration times and patient length of stay (LOS) or consumption of other resources. By not taking these uncertainties into account, the hospital could face serious service quality problems that could generate unexpected costs. European hospitals su er numerous reforms required by the authorities in order to improve the cit-izens' quality of life, especially in Belgium and France .Furthermore, a continually growing demand for medical care, an aging population, requirements from better informed patients, and the evolution of the pathologies are to be added to these structural challenges, and consequently increase the complexity of the problem. Health care institutes used to have less managerial control. Nowadays, hospitals are under intense nancial pressure where cost management and health care service are equally important. Hospital managers feel obliged to take a look at their resource manage-ment, because they are challenged to deliver high quality care with limited resources. The operating theatre, consisting of operating rooms (ORs) and recovery rooms, is one of the most important and critical resources. The entire OR department is a key hospital resource, as 70% of all hospital admis-sions are meant for surgery procedures and this accounts for 40% of the total expenses of a hospital [Litvak and Long, 2000]. The surgical scheduling processes appear to be very complex tasks, because of the multitude of stakeholders (patients, OR managers, surgeons and nurses) pursuing di erent objectives and interests. Therefore, health care managers are looking for the largest cost and revenue factors in hospitals, which are publicly known to be the sta and the operating theatre respectively [Litvak and Long, 2000], [Guinet and Chaabane,

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2003], [Belien et al., 2007]. The two problems, the surgery scheduling problem and the personnel rostering problem are closely linked in an operating theatre environment [Beli•en and Demeulemeester, 2008]. We consider the surgery scheduling problem as a weekly design for the operating theatre where surgeries of the elective patients are assigned to time blocks and surgeons with speci c specialties. It is not su cient to only incorporate the daily physician demand as a constraint in a surgery problem formulation. The actual assignment of the physicians to time instances should be scheduled according to surgery assignment.

Physician rostering is a special case of sta scheduling and rostering, which has been excellently reviewed by [Ernst et al., 2004]. Physicians generally have more complex working agreements and contract rights than general sta ng problems. In practice they also have insight in the planning and rostering phase in order to choose or give preference to speci c time instances. Literature identi es two methods of planning, namely cyclic and acyclic planning. Acyclic or ad-hoc rosters must be rebuilt every planning period because physicians may have di erent work rosters every week. Cyclic planning creates patterns for physicians, with or without a weekly rotation [Carter and Lapierre, 2001]. The complete planning process for the physicians consists of three stages. In the strategic phase policy de-cisions are taken by the management of the hospital. These decisions include restrictions on monthly working hours, level of preferences, maximal amount of overtime hours, etc. The tactical phase is the actual scheduling phase wherein the roster for the physicians is created. The roster indicates the working days and the days o and also handles personal preferences and vacation of personnel. Finally, the operational phase adopts a day-to-day focus and assigns di erent tasks to physicians. The contribution of this paper is twofold. We propose a stochastic programming approach for the IPSSP, and an exact L-shaped algorithm to solve this stochastic integer programming problem. This exact method is able to create an optimal solution for up to 1,000 scenarios per problem instance or only 2 realizations per surgery duration. Therefore the Sample Average Approximation (SAA) tech-nique is used to allow us to provide optimal results for the Stochastic IPSSP (SIPSSP) for real life instances and cases with millions of scenarios. Secondly, we discuss the computational enhancements that bene t the solution times of the exact method. The outline of this paper can be summarized as follows. In section 2 the Stochastic Integrated Physician and Surgery Scheduling Problem (SIPSSP) is presented and linked with literature. The stochastic programming formulation with recourse is given and explained. Section 3 applies the L-shaped method on the SIPSSP by formulating the two stage stochastic program with overtime minimization recourse function. We propose several computational enhancements for the exact L-shaped method in order to tighten bounds, reduce subproblem computation time and reduce master problem computation time in section 4. In section 5 the Sample Approximation technique is introduced in order to achieve approximate solutions.

Methods

The Integrated Physician and Surgery Scheduling Problem

Problem formulation

The IPSSP [Van Huele and Vanhoucke, 2014a] is formulated as follows: given are a set of surgeries S(index s = 1; 2; : : : ; jSj) to be scheduled on a weekly basis, consisting of a set of days D (index d = 1; 2; : : : ; jDj). These surgeries with duration ds have to be assigned to a speci c operating room (OR) from the set of ORs R (index r = 1; 2; : : : ; jRj) and on a certain moment in time T (index t = 1; 2; : : : ; jT j) and a speci c day. The physicians P (index p = 1; 2; : : : ; jP j) that perform the surgeries are rostered according to their preferences, skills and holidays. The assignment of surgeries depends on the arrival time of a patient [ESs; LSs] de ned by the earliest start and latest start of a surgery and of the daily bed availability Bd, bearing in mind that every surgery has a certain length of stay (LOS) bs. The rostering of the surgeons is subject to their preferences towards weekly working hours, maximal daily workload, skills and unavailable time. The objective is to minimize the overtime for all OR-days.For the physician part, the supply and demand constraints are inevitable and instigated by the number of patients that have to be scheduled that week. The supply is the number of physicians available to perform surgery that week. Physician's roster preferences are translated into minimum and maximum working hours per week, respectively de ned by Npmin and Npmax. Personal aversions of physicians for speci c time instants t at a speci c day d are inclined by the set Opdt. Some physicians are not allowed to do more than Qp surgery hours per day. We also assume that the planned surgery cases can all be scheduled within the week schedule window.

Applying the L-shaped method on SIPSSP

In the L-shaped method the problem is decomposed into one master and j j subproblems, one for each scenario. At iteration n of the L-shaped method, the master problem is solved to obtain feasible rst-stage decision variables, xn. The second stage problem is evaluated using the rst stage variables for each scenario. The dual information from the second stage solution are used to form a cut to be added at the rst stage problem.

Computational enhancements

We have established an L-shaped method that enables e cient exact solutions for the stochastic IPSSP for relatively small instances and a small set of scenarios. In this section, several computational en-hancements are added to further decrease computation time. These are LP warm starting, use of tighter lower bounds, Hamming distance trust region, multi-cut clustering and sampling approxima-tion. Below, we explain these in more detail and in the next section we provide the results of our computational tests on the e ectiveness of our enhancements.

LP-warm start

Rounding the linear programming relaxation of the stochastic program can be used to initiate the branch-and-bound search process. The following steps are used in our rounding heuristic:

This integer solution is then used as a MIP start in CPLEX. It may be infeasible or even incomplete. CPLEX 12.4 easily supports the use of warm starting in a branch and cut optimization procedure. The optimality cuts generated by solving the LP solution are valid for the integer program, given the continuous second-stage problems. These optimality cuts provide valuable information on the shape of the recourse function. However, the integer master problem bulks up in size and computation time at each L-shaped iteration as more cuts are present. Fortunately, CPLEX 12.4 o ers the possibility of using the so-called lazy constraint callback.

Results

Computational results

In this section the L-shaped method along with the computational enhancements of section 4 will be tested. In the experiment some parameters are xed. We consider the case of 3 operating rooms where 30 surgeries have to be scheduled by 7 physicians. These surgeries have to be scheduled on a weekly basis (5 days) where it is possible to roster a surgery every hour. The average surgery durations davgs are generated according to a uniform distribution. Length of stay (LOS) is sampled from a uniform distribution U[1; 3]. The duration d!s of the second stage problem is the stochastic parameter with a discrete lognormal distribution ln N (; 2) with mean

= davgs and variance 2 = 2. The surgery duration data was gathered from the Maria Middelares Hospital in Sint-Niklaas, Belgium, more speci cally from the specialities orthopedic, stomatologic and neurologic surgery for all physicians. Table 1 summarizes the data of the hospital that resulted in the suggested distribution. The summary includes the scale, the 95% CI on the scale and the arithmetic variance for every specialisation. Fitting these 1300 data samples resulted in a lognormal distribution. We have not taken into account any predictive error in the duration of the cases ([Dexter et al., 2013a], [Dexter et al., 2010]). (**Table 1**)

Table 1: *Surgery specialties data*

Specialisation	Scale	Variance	Ci on scale	# Surgeries (samples)
Neurological	0.5395833	0.0993056	[0:702; 0:848]	207
Stomatology	0.4395833	0.0840278	[0:585; 0:678]	315
Orthopedic	0.4652778	01:53	[0:636; 0:701]	852

We only consider the rst 10 surgeries to have a stochastic duration, all others have a deterministic duration between the uniform interval U[3; 5]. Let j j be the number of realizations per surgery du-ration sampled from the lognormal distribution with probability P rob for every realization 2 and the probability of a scenario ! is Prob! = Q 2 ( ) Prob . Overtime cost is set at e500 per hour. Each day consists of 12 hours, from which 8 hours are regular OR hours and 4 hours overtime.

When considering the 10 surgeries with 2 realizations, we have 210 = 1024 scenarios to consider. If the number of realizations are augmented to 4, yielding 410 = 1; 048; 576 scenarios, one quickly comes to the conclusion that considering variability in the solution approach forces the number of scenarios to grow exponentially. To limit this growth, we do not only limit the number of surgeries to contain the variability, but also the number of realizations per surgery. When considering the lognormal distribu-tion with mean 3 and variance 2, only 2 realizations of the surgery hours are taken into consideration to form the scenarios.

All experiments were performed on the Stevin Supercomputer Infrastructure (Gengar) provided by Ghent University. The cluster contains 94 computing nodes (IBM HS 21 XM blade), each of which contains a dual-socket quad-core Intel Xeon L5420 (Intel Core microarchitecture, 2.5 GHz, 6 MB L2 cache per quad-core chip), thus 8 cores / node with 16 GB RAM. The algorithm was written in C++ and linked with the CPLEX 12.6 optimization library.

Simulation results

In this section, we will investigate the necessity of stochastic programming for SIPSSP instances. Justifying the e ort of modeling a stochastic program can be proven by comparing simulation results of the stochastic solution (based on SAA) and the expected value problem (EV).

Let us de ne the solution of the expected value problem as xEV and the solution of the stochastic problem as xSAA. Furthermore, let us de ne the solution value zEV corresponding to the optimal solution of the deterministic model xEV . Analogously, let us de ne zSAA as the solution value of the sampling method SAA over the stochastic L-shaped method for solution xSAA. In the previous section, we have proven the upper bound of the sampling method SAA (xSAA) to be better than the solution of the expected value problem xEV for sample size greater than 5 for this instance. Also, sample size of N = 250 yields 1% optimal results.

Figure 1 shows a boxplot for both methods (SAA and EV) and for every variance step in the lognor-mal function lnN (; 2). For every step, it is possible to compare the means of the two boxplots. The means and standard deviations is written in parenthesis underneath the boxplots for both methods. The stochastic solution (SAA) shows mildly better results than the deterministic variant (EV), taking into account our representation of uncertainty in the durations of the surgery cases. However, the di erence for small variations is insigni cant. In **Figure 1** we can see a clear rise in overtime cost for both methods when variance rises. A second observation is that the variance considering the overtime costs for the expected value problem is higher than for the SAA method, which signi es robustness when solving the problem stochastically. This can be seen by the whiskers of the boxplot. It is imperative to note that the mentioned standard deviation of the lognormal distribution in this research is vastly di erent from what resulted from the research of [Dexter et al., 2013b] and [Dexter et al., 2013a] in which the average of standard deviations typically is 0.3 for 354 scheduled procedures, whereas the data in this research from orthopedic cases shows a standard deviation of 1.2.

Discussion

Our observations on the impact of using stochastic optimization methods for minimizing overtime in operating

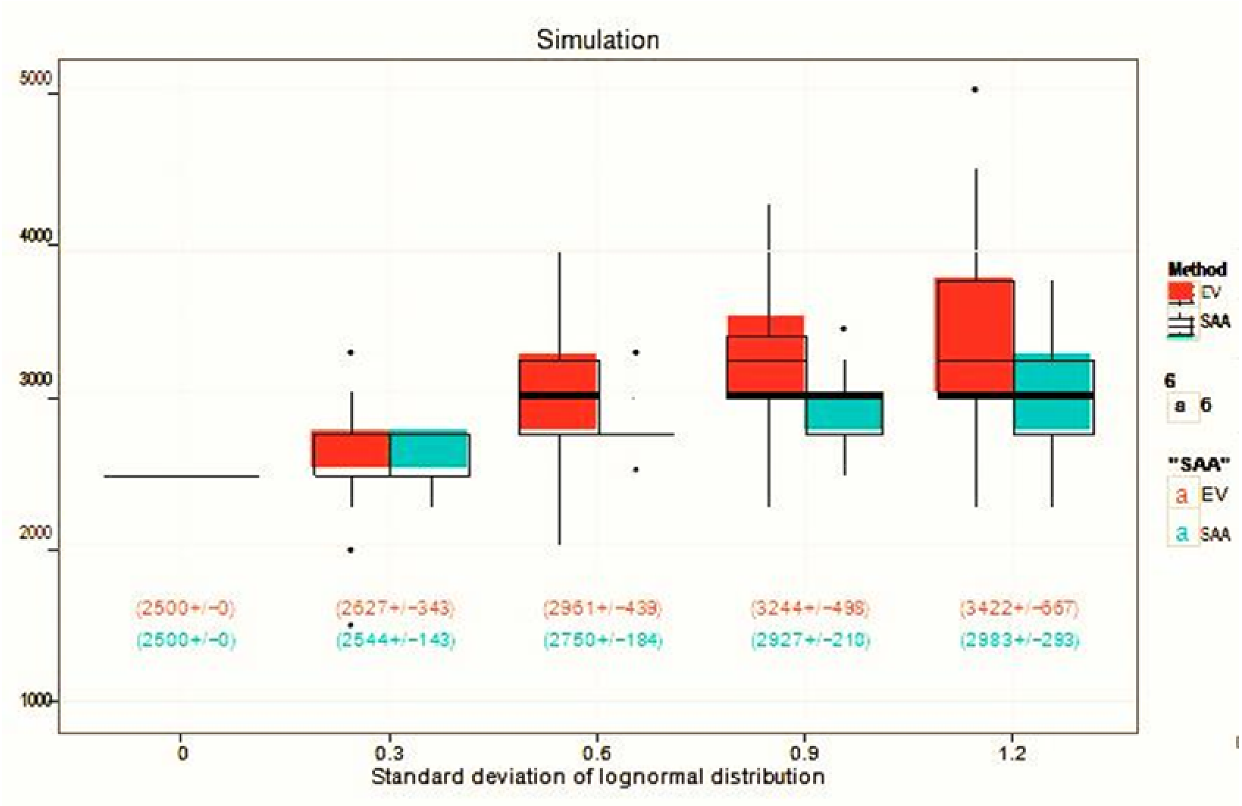

Figure 1: *Simulation on SAA and EV solution.*

theatres are largely dependent on the variation within surgery duration data. Due to the fact that the surgery duration data from the Neurological, Stomatology and Orthopedic specialties do not contain separate procedure speci c nor physician speci c data and are consequently pooled together ensures a rise in the resulting variance of the durations. Our research shows that for higher variability, the use of stochastic optimization methods should be considered. Our model considers uncertainty only in the duration of the surgery, while other model parameters can be the cause of variability in an OR. For example, the length of stay parameter can be subject to change and can therefore be considered in the stochastic model as a stochastic parameter in future research. In this paper, we addressed the uncertainty by creating schedules that are robust in terms of quality of the solution. Alternatively, creating robust schedules in terms of the solution itself in which the starting times of the surgeries are considered as robust may be preferred in some cases. Di erent optimization techniques should be considered then.

Conclusion

We have applied an exact optimization technique, called the L-shaped method to solve the stochastic case of the integrated physician and surgery scheduling problem. The stochastic integrated physician and surgery scheduling problem di ers over the deterministic case by the variability over the duration of the surgeries. This SIPSSP can be formulated as a two stage stochastic program with recourse func-tion. In the rst stage of the program, the assignment of the surgeries to the physicians and OR-day time instances with respect to the expected values of the respective surgeries. In a second stage the expected overtime of every scenario is evaluated and eventually minimized. This two-stage stochastic program with integer recourse can be solved with the L-shaped

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method. This exact optimization technique is based upon Benders decomposition and is one of the most common techniques for these type of problems. The L-shaped method can be strengthened with computational enhancements. The combination of an exact algorithm for solving the subproblem, using trust regions, tighter lower bounds and warm starts in the MIP based on the LP relaxation ensured a computational reduction in solution time of more than 50%. Unfortunately, solving small SIPSSP problems, even with an MIP gap of 1%, is intractable when the number of realizations and thus the number of scenarios rises to more than 210. To alleviate this problem, we introduced a sampling method, called the Sample Average Approximation method that obtains high quality solutions without having to evaluate every scenario. These sampling techniques come at a certain price. Upper and lower bounds are found with a corresponding variance and thus yield -optimal solutions. However, optimality gaps of 1% can be achieved for instances with more than 630 scenarios in an hour frame in our computational results.

Acknowledgement

None.

Conflict of Interest

None.

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