DEMAND-ORIENTED TRADE EQUILIBRIUM OF MULTI-NATIONAL ECONOMIES

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ABSTRACT

This paper proposes a new international trade model, the demand-oriented trade model, to examine the interdependence and interaction of multi-country economies. It reflects realistic considerations in international trade practice: intermediate input, mobile factors across countries, intra-industry trade, and technology differences. The model emphasizes that taste-based internationalized demand plays an important role in the equilibrium of multi-national economies. This paper provides an approach to process demand-oriented trade analysis, which is a weak part in existing trade models. It introduces an account matrix of international trade to explore the production-trade structure of multi-national economies. The study demonstrates that there are five basic equilibriums in international trade and production: the factor resource constraint equilibrium; the intermediate input output equilibrium of production; the equilibrium of final goods transacted from the domestic supply to the internationalized demand; the price equilibrium; and the reciprocal equilibrium between the internationalized demand and the domestic factors. All of them, with reductions, exist both in the Ricardo model and in the Heckscher-Ohlin model. The higher dimension (k factors, n sectors, and m countries) model of this paper is an "uneven number" model, which allow the number of factors to be not equal to the number of commodities. This paper also introduces the matrix of factor content of trade flows and a new measurement for factor abundance based on real imports and exports.

INTRODUCTION

Multi-national economies can be considered as a system of interdependence and interaction processes by trade and factor mobility. A direct interdependence between two countries arises whenever the output or export of one nation becomes an input or import of another.

Both the Ricardo model and the Heckscher-Ohlin model emphasize that the two countries' economies are an integrated production-trade system and on that the economic equilibrium of two countries is the integrated production-trade equilibrium.

The Heckscher-Ohlin model and the Heckscher-Ohlin theorem demonstrated the basis for comparative advantage and the effects that trade had on factor earnings in the two countries. Many efforts have been made to extend the Heckscher-Ohlin model to explain the reality of world trade for the last four decades. There has been considerable progress in the literature on the studies in intermediate goods, mobile factors, intra-industry trade, effects of different technologies, and higher dimension application models. Vanek (1968) proposed a multi-factor and multi-good model, which is often referred to the HOV model. This model has led to a lot of empirical researches (such as Leamer, 1980; Trefler, 1993; 1995). Davis and Weinstein (2001b) estimated technology matrix across the OECD countries and used it to test the HOV equation. Other literatures have studied the influence of public intermediate goods on the fundamental theorem in traditional trade theories (Kahn, 1980; Okamoto, 1985; Altenburg, 1987).

This paper introduces a new international trade model, the demand-oriented trade model. It incorporates intermediate input, factor mobility, intra-industry trade, consumption taste, and technology differences to form an integrated trade model. This paper introduces an account matrix of international trade to describe the trade flows and factor mobility flows in multi-country economies. It illustrates that when trade networks multi-national economies, there are five basic structural equilibriums. The first one is the equilibrium between factor mobility and output with both domestic and international factor mobility. The second is the equilibrium among intermediate input, final goods, and output, which is presented by a traditional Leontief multi-region input-output function. The third is the trade equilibrium of final goods, which depicts the transaction equilibrium from domestic supply to internationalized demand. The fourth is the factor-demand reciprocal equilibrium, which demonstrates a direct relationship between domestic factor and internationalized demand. The fifth is the international price equation, which shows how factor price determines commodity price when considering intermediate input and factor mobility across countries. The study shows that both the Ricardo model and the Heckscher-Ohlin model are special reduced cases of the demand-oriented trade model.

This paper also provides an analytical approach to prove the Ricardian law of comparative advantage with the help of demand-oriented trade analysis. The study proposes the higher dimension (k factors, n sectors, and m countries) demand-oriented trade model to simulate the trade connections of the real world. The paper introduces the matrix of factor content of trade flows and a measurement for factor abundance based on real imports and exports.

ACCOUNT MATRIX OF INTERNATIONAL TRADE

We begin by proposing an account matrix of international trade in this section, to give a general picture of the trade flows and factor mobility in multi-national economies. The Account Matrix of International Trade (AMIT) is a square matrix to depict domestic trade flows, international trade flows, intermediate trade flows, final goods trade flows, and international mobile factors, for multi-national economies. It is very similar to the SAM (social account matrix) in nation-wide economic analysis, but it is in the framework of multi-national economies. It consists of row and column accounts that represent the different sectors of a multi-national economy at the desired level of disaggregating. By convention, each account in the AMIT is represented by one row and one column of the table; each cell represents a physical quantity imported (or expenditure) by the column account and a quantity exported (or income) by the row account. The underlying principle of double-entry accounting requires that the total revenue of exports (row total) must equal the total expenditure of imports (column total) for each account in the AMIT. The AMIT is a visual framework for displaying multi-country, multi-sector, and multi-factor economic structure that integrates input-output flows, import-export flows, and factor flows into a comprehensive and consistent dataset. Once an AMIT is constructed, it provides a static image, or a snapshot, of the multi-country production-trade structure.

Table 1. Account Matrix of International Production and Trade

			Intermediate Output				Final Goods					
			Country H		Country F		Country H		Country F		Supply	Output
			1	2	1	2	1	2	1	2	y	x
Intermediate Input	Country H	1	x_{11}^{HH}	x_{12}^{HH}	x_{11}^{HF}	x_{12}^{HF}	y_{11}^{HH}	0	y_{11}^{HF}	0	y_1^H	x_1^H
		2	x_{21}^{HH}	x_{22}^{HH}	x_{21}^{HF}	x_{22}^{HF}	0	y_{22}^{HH}	0	y_{22}^{HF}	y_2^H	x_2^H
	Country F	1	x_{11}^{FH}	x_{12}^{FH}	x_{11}^{FF}	x_{12}^{FF}	y_{11}^{FH}	0	y_{11}^{FF}	0	y_1^F	x_1^F
		2	x_{21}^{FH}	x_{22}^{FH}	x_{21}^{FF}	x_{22}^{FF}	0	y_{22}^{FH}	0	y_{22}^{FF}	y_2^F	x_2^F
							u_1^H	u_2^H	u_1^F	u_2^F		Factor
Factor Input	Country H	L	v_{11}^{HH}	v_{12}^{HH}	v_{11}^{HF}	v_{12}^{HF}	z_{11}^{HH}	z_{12}^{HH}	z_{11}^{HF}	z_{12}^{HF}		L^H
		K	v_{21}^{HH}	v_{22}^{HH}	v_{21}^{HF}	v_{22}^{HF}	z_{21}^{HH}	z_{22}^{HH}	z_{21}^{HF}	z_{22}^{HF}		K^H
	Country F	L	v_{11}^{FH}	v_{12}^{FH}	v_{11}^{FF}	v_{12}^{FF}	z_{11}^{FH}	z_{12}^{FH}	z_{11}^{FF}	z_{12}^{FF}		L^F
		K	v_{21}^{FH}	v_{22}^{FH}	v_{21}^{FF}	v_{22}^{FF}	z_{21}^{FH}	z_{22}^{FH}	z_{21}^{FF}	z_{22}^{FF}		K^F

Table 1 is such a hypothetical account matrix of international trade, which describes a production-trade system of two factors (capital K and labor L), two countries (H and F), and two goods (1 and 2).

There are four phases in the table. Phase I (the left-bottom corner in the table) depicts the factor mobility of two countries. The Heckscher-Ohlin model assumes that factors are perfectly mobile within each country and not mobile internationally. This paper assumes that factors may move both domestically and internationally. But these international movements might be incomplete because of various barriers. The factor flows in Phase I can be described by a matrix

$$V = \begin{bmatrix} v_{L1}^{HH} & v_{L2}^{HH} & v_{L1}^{HF} & v_{L2}^{HF} \\ v_{K1}^{HH} & v_{K2}^{HH} & v_{K1}^{HF} & v_{K2}^{HF} \\ v_{L1}^{FH} & v_{L2}^{FH} & v_{L1}^{FF} & v_{L2}^{FF} \\ v_{K1}^{FH} & v_{K2}^{FH} & v_{K1}^{FF} & v_{K2}^{FF} \end{bmatrix} \tag{1}$$

where V is the factor mobility matrix, its typical element v_{sj}^{hf} is the quantity of factor s from country h input in sector j in country f; $h,f=H,F$; $s= L,K$; and j=1,2.

Phase II (the left-upper corner of the table) shows the trade flows of intermediate goods. It can be described by the following matrix,

$$X = \begin{bmatrix} x_{11}^{HH} & x_{12}^{HH} & x_{12}^{FH} & x_{12}^{FH} \\ x_{21}^{HH} & x_{22}^{HH} & x_{12}^{HH} & x_{12}^{HH} \\ x_{12}^{HF} & x_{12}^{HF} & x_{11}^{FF} & x_{12}^{FF} \\ x_{12}^{HF} & x_{12}^{HF} & x_{21}^{FF} & x_{22}^{FF} \end{bmatrix} \tag{2}$$

where X is the trade flow matrix of intermediate goods; its typical element x_{ij}^{hf} is the quantity of output i in country h used in sector j in country f ; $h,f=H,F;$ i,j=1,2.

Phase III (the right-upper corner of the table) illustrates the trade flows of final goods, which can be denoted as

$$Y = \begin{bmatrix} y_{11}^{HH} & 0 & y_{11}^{HF} & 0 \\ 0 & y_{22}^{HH} & 0 & y_{22}^{HF} \\ y_{12}^{FH} & 0 & y_{11}^{FF} & 0 \\ 0 & y_{22}^{FH} & 0 & y_{22}^{FF} \end{bmatrix} \tag{3}$$

where Y is the trade flow matrix of final goods; its typical element y_{ii}^{hf} is the quantity of final goods i made in country h exported to country f.

The pattern of final goods trade flows of matrix Y is different from the pattern of intermediate trade flows of matrix X in that all elements y_{ij}^{hf}=0 for i not equal to j, because there is no sector category in the consumption of final goods. A country, as whole, is in one count of the consumptions of final goods.

We clarify the terms used here. We refer the final goods produced in a country to the supply of final goods (briefly as supply) and refer the final goods consumed in a country to the demand of final goods (briefly as demand).

The row sum of matrix Y is the supply of final goods, i.e.

$$y = Yl$$

where $$y = \begin{bmatrix} y_1^H \\ y_2^H \\ y_1^F \\ y_2^F \end{bmatrix} \tag{4}$$

y is the vector of the supply of final goods for two countries; its typical element y_i^h is the supply of final goods i in country h; l is the identical vector, and all its elements are 1.

The column sum of matrix Y is the demand of the final goods, i.e.

$$u = Y^T l$$

where $$u = \begin{bmatrix} u_1^H \\ u_2^H \\ u_1^F \\ u_2^F \end{bmatrix} \tag{5}$$

u is the vector of demand of final goods; its typical element u_i^h is the quantity of demand of good i in country h.

In autarky, the supply of final goods equals the demand for the final goods. After trade, the supply is still the domestic supply, but the demand is the internationalized demand. This is an important characteristic in an open economy.

The expression of final goods flows Y is the critical foundation for all the following discussions of this paper. To make it clear, we explain now it in another way. The supply of final goods can be divided into two parts. One part is the final goods produced in the host country and consumed in the host country. The other part is the final goods produced in the host country and exported to the foreign country. The vector of supply in a country, h, can be expressed as.

$$y^h = y^{hh} + y^{hf} = \begin{bmatrix} y_1^{hh} \\ y_2^{hh} \end{bmatrix} + \begin{bmatrix} y_1^{hf} \\ y_2^{hf} \end{bmatrix} \qquad (6)$$

where y^h is the vector of final goods of country h; y^{hh} is vector of supply of final goods produced in country h and consumed in country h; and y^{hf} is vector of final goods produced in country h and exported to country f; $h=H,F$.

The matrix of final goods flows actually is arranged by

$$Y = \begin{bmatrix} diag(y^{HH}) & diag(y^{HF}) \\ diag(y^{FH}) & diag(y^{FF}) \end{bmatrix} = \begin{bmatrix} y_1^{HH} & 0 & y_1^{HF} & 0 \\ 0 & y_2^{HH} & 0 & y_2^{HF} \\ y_1^{FH} & 0 & y_1^{FF} & 0 \\ 0 & y_2^{FH} & 0 & y_2^{FF} \end{bmatrix} \qquad (7)$$

where $diag(y^{hf})$ is the diagonal matrix of vector y^{hf}, its diagonal elements are the elements of vector y^{hf}, all non-diagonal cells are zero.

International trade can be considered as a substitution for international mobility of factors. Phase IV (in the right-bottom corner of the table) shows such substituted factor flows, which illustrates the factor content of trade flows in phase III. We denote the factor content of trade flows as

$$Z = \begin{bmatrix} z_{L1}^{HH} & z_{L2}^{HH} & z_{L1}^{HF} & z_{L2}^{HF} \\ z_{K1}^{HH} & z_{K2}^{HH} & z_{K1}^{HF} & z_{K2}^{HF} \\ z_{L1}^{FH} & z_{L2}^{FH} & z_{L1}^{FF} & z_{L2}^{FF} \\ z_{K1}^{FH} & z_{K2}^{FH} & z_{K1}^{FF} & z_{K2}^{FF} \end{bmatrix} \qquad (8)$$

where Z is the substituted factor flows matrix; its typical element z_{sj}^{hf} is the substituted amount of factor s from country h input in sector j in country f, which is the factor content behind trade flow y_{ii}^{hf}.

We will discuss the properties of the matrix of substituted factor flows and its computation in detail in a later section of this paper.

DEMAND-ORIENTED TRADE MODEL

We will derive the demand-oriented international trade model in this section, based on the structure of trade flows and mobile factors shown in Table 1.

The proposed demand-oriented international trade model is comprised of the five equations below.

Intermediate Input Output function

The row direction of phases II and III in table 1 shows the distribution equilibrium of output. This equilibrium can be expressed by

$$x = Ax + y \tag{9}$$

where

$$A = \begin{bmatrix} a_{11}^{HH} & a_{12}^{HH} & a_{12}^{FH} & a_{12}^{FH} \\ a_{21}^{H} & a_{22}^{H} & a_{12}^{HH} & a_{12}^{HH} \\ a_{12}^{HF} & a_{12}^{HF} & a_{11}^{FF} & a_{12}^{FF} \\ a_{12}^{HF} & a_{12}^{HF} & a_{21}^{FF} & a_{22}^{FF} \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} x_1^H \\ x_2^H \\ x_1^F \\ x_2^F \end{bmatrix}$$

x is a vector of output; its typical element x_i^h is the output of good i in country h; A is a coefficient matrix of intermediate input in a two-country framework; it is defined by

$$A = X[diag(x)]^{-1}$$

where *diag(x)* is the diagonal matrix of vector x, its main diagonal elements are the elements of vector x and all other elements are zeros.

Equation (9) is a typical Leontief multi-regional input-output equilibrium. The international production-trade system should include the multi-national input-output processes of production.

The Trade Function of Final Goods

If we introduce an import coefficient matrix:

$$C = Y[diag(u)]^{-1}$$

where *diag(u)* is the diagonal matrix of vector *u*, its main diagonal elements are the elements of vector *u* and all other elements are zeros.

The trade transaction equilibrium of final goods in Phase III can be presented as

$$y = Cu \tag{10}$$

where

$$C = \begin{bmatrix} c_{11}^{HH} & 0 & c_{11}^{HF} & 0 \\ 0 & c_{22}^{HH} & 0 & c_{22}^{HF} \\ c_{11}^{FH} & 0 & c_{11}^{FF} & 0 \\ 0 & c_{22}^{FH} & 0 & c_{22}^{FF} \end{bmatrix} \tag{11}$$

C is the import coefficient matrix of final goods; its typical element c_{ii}^{hf} signifies the amount of good *i* that is made in country *h* and consumed in country *f* per the demand of the good in country *f*.

One characteristic of matrix *C* is that each column sum 1, i.e.,

$$C^T l = l \tag{12}$$

where *l* is an identity vector.

Matrix *C* reflects the international taste pattern or import pattern of the two countries. And equation (10) demonstrates how the domestic supply of final goods is transacted into the internationalized demand after trade for a given taste pattern.

Factor Resource Constraint Function

Assuming full-employment conditions, factor resource constraint can be presented as:

$$v = Bx \tag{13}$$

where

$$v = \begin{bmatrix} L^H \\ K^H \\ L^F \\ K^F \end{bmatrix}, \qquad B = \begin{bmatrix} b_{L1}^{HH} & b_{L2}^{HH} & b_{L1}^{HF} & b_{L2}^{HF} \\ b_{K1}^{HH} & b_{K2}^{HH} & b_{K1}^{HF} & b_{K2}^{HF} \\ b_{L1}^{FH} & b_{L2}^{FH} & b_{L1}^{FF} & b_{L2}^{FF} \\ b_{K1}^{FH} & b_{K2}^{FH} & b_{K1}^{FF} & b_{K2}^{FF} \end{bmatrix}$$

v is the vector of factor inputs; L^h is the labor resource in country h; K^h is the capital resource in country h; B is the coefficient matrix of primary inputs, its typical element b_{sj}^{hf} signifies the amount of input of factor s from country h to sector j in country f per output j in country f.

A new feature of this function is that it may include mobile factors across countries.

Demand-Oriented Factor Equilibrium

Equation (9) can be rewritten as

$$x = (I - A)^{-1} y \tag{14}$$

Substituting equation (10) into equation (14) yields

$$x = (I - A)^{-1} \alpha u \tag{15}$$

where I is identity matrix, its main diagonal elements are 1 and all other elements are zeros.

It shows the demand-oriented output equilibrium.

Substituting equation (14) into equation (13) yields

$$v = B(I-A)^{-1}y \tag{16}$$

It illustrates the supply-oriented factor equilibrium.

Further more, substituting equation (15) into equation (13) yields

$$v = B(I-A)^{-1}Cu \tag{17}$$

It demonstrates the demand-oriented factor equilibrium.

When the number of factors equals the number of commodities (the so-called even number case) and when $B(I-A)^{-1}C$ is nonsingular, equation (17) can be reversed as

$$u = C^{-1}(I-A)B^{-1}v \tag{18}$$

Equation (17) and (18) establish the reciprocal relationships between the domestic factor inputs and the internationalized demand. The factor requirement by equation (17) is demand-oriented, and the demand equilibrium by equation (18) is endowment-factor-oriented. The factor requirement in equation (17) is the total requirement of factors, because $B(I\text{-}A)^{-1}$ is the coefficient matrix of the global total requirement; it includes both primary factor requirements and intermediate factor requirements (see Feenstra, 2004, 37; Davis & Weinstein, 2001a). Differencing from the total factor requirement used by Davis and Weinstein (2001a), which is a domestic total requirement, the global total requirement of factors in this paper may deal with international intermediate input and international factor movements.

International Price Function

Along the column direction from Phase II through Phase I in table 1, we can find the components of the costs of production of each sector. The total value of any output should be equal to the total value of factor inputs in it plus the total value of intermediate inputs in it under the zero-profit conditions. This can be calculated by

$$p^T X + w^T V = p^T diag(x) \tag{19}$$

where

$$w = \begin{bmatrix} w_1^H \\ w_2^H \\ w_1^F \\ w_2^F \end{bmatrix}, \qquad p = \begin{bmatrix} p_1^H \\ p_2^H \\ p_1^F \\ p_2^F \end{bmatrix}$$

diag(x) is a diagonal matrix in which the elements of the vector of output *x* appear in its main diagonal cells and zeros appear in the other cells; *p* is the vector of price; its typical element p_i^h is the price of good *i* in country *h*; *w* is the vector of factor price; its typical element w_s^h is the price of factor *s* in country *h*.

The price in equation (19) is an international price, in which both factor price and commodity price are measured by a single currency, such as USD or EUR.

Matrixes X and V can be expressed as

$$X = A \cdot diag(x) \tag{20}$$

$$V = B \cdot diag(x) \tag{21}$$

Substituting equations (20) and (21) into (19) yields,

$$p^T A \cdot diag(x) + w^T B \cdot diag(x) = p^T \cdot diag(x) \tag{22}$$

It can be reduced as

$$p^T A + w^T B = p^T \tag{23}$$

Transposing it yields

$$p = (I - A^T)^{-1} B^T w \tag{24}$$

This is the price equation both with intermediate input and with primary factor input. The price in equation (24) is an absolute price rather than an index price. One property of the price is that the total value of final goods valued by commodity price equals the total value of factor inputs valued by factor price, i.e.,

$$p^T y = w^T v \tag{25}$$

The proof of it is given in Appendix A in the end of this paper.

Demand-Oriented Trade Model

The intermediate input output equation (9), factor resource constraint function (10), trade function (13), demand-oriented factor requirement (17), and price function (24), as whole, comprise the demand-oriented trade model, which demonstrates the equilibriums of international production-trade from various angles. The basic assumption for the demand-oriented trade model is that commodities are produced under constant returns to scale.

The demand-oriented trade model proposed in this paper is a practical and theoretical extension of the Heckscher-Ohlin model. It uses the trade function of final goods to establish the trade connection; realizes the Linder consumption taste in the model structure; reflects intra-industry trade both in intermediate good trade and in final goods trade; includes mobile factors across countries; and includes different technologies across countries. The model emphasizes that consumption tastes play an important role in the equilibrium of multi-national economies. Differing from the Heckscher-Ohlin theorem that predicts trade direction from factor earnings and factor abundance, the demand oriented trade model predicts the requirement of domestic factors from taste-based international demand and describes the structure interdependences of the multi-national production-trade system. Like the Leontief input-output analysis, the demand-oriented trade model can serve as a policy simulation tool.

The economy is marked-oriented; international trade is demand-oriented. The demand-oriented trade model implies that the production and the supply of a country is oriented both by its domestic demand and by international demand. The equilibrium of the international economy is the demand-supply equilibrium of all trading countries; and the multi-national demand-oriented development can benefit all the countries.

The analysis in this section is summarized in the following proposition.

Proposition - In open economic system, the output of any country in the system is oriented by the supply of final goods, i.e. $x=(I-A^T)_{-1}y$, which is driven by the internationalized demand of the final goods, i.e. $y=Cu$. The domestic factor requirement of all countries are finally allocated by the internationalized demand of whole system, i.e. $v=B(I-A^T)^{-1}Cu$.

DISTRIBUTION OF FACTOR CONTENT OF TRADE FLOWS

The factor content of trade flows or "substituted" factor flows accompanying trade flows, mentioned in section 2, can be expressed by:

$$Z=B(I-A)^{-1}Y \tag{26}$$

or

$$Z=B(I-A)^{-1}C\cdot diag(u) \tag{27}$$

where $diag(u)$ is the diagonal matrix of vector u.

It has the following three properties:

The first property is that the row sum of the matrix of the substituted factor mobility is equal to the factor vector, i.e.,

$$Zl=v \tag{28}$$

The proof of this property is given in Appendix B at the end of this paper.

The second is that the column sum of the matrix of the substituted factor mobility valuated by factor price is equal to the column sum of the matrix of final goods valued by commodity price, i.e.,

$$w^T Z = p^T Y \tag{29}$$

It is proved in Appendix C at the end of this paper.

The last one is that when trade is zero, *(y=u and C=I)*, the matrix of the substituted factor flow is equal to the matrix of real factor flows. i.e.,

$$V = Z \tag{30}$$

Appendix D shows the proof of this property.

Phase I in table 1 can be regarded as a block of GDP content of factor inputs in a two-country framework; its monetary sum by factor earning rate is the GDP of the two countries from income approach. Phase III is the block of GDP content of trade flows; its monetary sum by commodity price is the GDP of two countries from expenditure approach. Phase IV is the block of factor content of trade. As we discussed above, the total monetary values of all elements in each block of the threes are the same, i.e.

$$w^T Z l = p^T Y l = w^T V l$$

ENHANCING THE HECKSCHER-OHLIN MODEL

"The Heckscher-Ohlin approach was primarily supply-oriented because it focused on factor endowments and factor intensities" (Appleyard & Field, 2001, 164). We will present the Heckscher-Ohlin model by using the framework of the demand-oriented model to enhance it on the trade-based demand analysis in this section.

Trade Flows of Inter-Industry Trade

The trade flows in equation (3) are intra-industry flows. The trade flows both in the Ricardo model and in the Heckscher-Ohlin model are inter-industry, because only the good intensive in the country's abundant factors (or the good produced in comparative advantage) should be traded. We can reduce the intra-industry trade flows in equation (3) into inter-industry trade flows only. Assume that country H is K-abundant; country F is L-abundant; and good 1 is K-intensive; good 2 is L-intensive. By these assumptions, country H would export good 1 and country F would export good 2. The trade flow matrix of equation (3) can now be reduced to inter-industry trade flow as,

$$Y=\begin{bmatrix} y_{11}^{HH} & 0 & y_{11}^{HF} & 0 \\ 0 & y_{22}^{HH} & 0 & 0 \\ 0 & 0 & y_{11}^{FF} & 0 \\ 0 & y_{22}^{FH} & 0 & y_{22}^{FF} \end{bmatrix} \tag{31}$$

Suppose that country *H* exports π of good 1 to country *F* and that the term of trade is α . Country *F* would export $\alpha\pi$ of good 2 to country H. The trade flow matrix above now can be written as,

$$Y=\begin{bmatrix} y_1^H-\pi & 0 & \pi & 0 \\ 0 & y_2^H & 0 & 0 \\ 0 & 0 & y_1^F & 0 \\ 0 & \alpha\pi & 0 & y_1^F-\alpha\pi \end{bmatrix} \tag{32}$$

The column sum of matrix *Y* is the internationalized demand,

$$u=Y^T l=\begin{bmatrix} y_1^H-\pi \\ y_2^H+\alpha\pi \\ y_1^F+\pi \\ y_2^F-\alpha\pi \end{bmatrix} \tag{33}$$

It is a general expression of demand for inter-industry trade pattern both for the Ricardo model and for the Heckscher-Ohlin model.

Gains-from-Trade Function

The gains from trade are evaluated by domestic prices in classical models. In autarky, supply equals demand. The difference between the demand and supply after trade is net import. The gain from trade by sectors can be expressed as

$$g_{sector} = \begin{bmatrix} g_1^H \\ g_2^H \\ g_1^F \\ g_2^F \end{bmatrix} = diag(p)\cdot(u-y) = diag(p)\cdot\left(\begin{bmatrix} y_1^H - \pi \\ y_2^H + \alpha\pi \\ y_1^F + \pi \\ y_2^F - \alpha\pi \end{bmatrix} - \begin{bmatrix} y_1^H \\ y_2^H \\ y_1^F \\ y_2^F \end{bmatrix}\right) = diag(p)\cdot\begin{bmatrix} -\pi \\ \alpha\pi \\ \pi \\ -\alpha\pi \end{bmatrix} = \begin{bmatrix} -p_1^H \\ \alpha p_2^H \\ p_1^F \\ -\alpha p_2^F \end{bmatrix}\pi \tag{34}$$

where g_{sector} is the vector of gains from trade by sectors of two countries; *diag(p)* is a diagonal matrix in which the elements of the vector of price p appear in the main diagonal cells and zeros appear in the other cells.

The gains from trade by country, or the gain-from-trade function, can be presented as:

$$g = \begin{bmatrix} g^H \\ g^F \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} g_1^H \\ g_2^H \\ g_1^F \\ g_2^F \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} -p_1^H \\ \alpha p_2^H \\ p_1^F \\ -\alpha p_2^F \end{bmatrix}\pi = \begin{bmatrix} -p_1^H + \alpha p_2^H \\ p_1^F - \alpha p_2^F \end{bmatrix}\pi \tag{35}$$

where g is the vector of gains from trade for two countries.

Enhanced Heckscher-Ohlin Model

The Heckscher-Ohlin model can be presented as a special case of the demand oriented trade model if we reduce the later by assuming (1) no intermediate input, i.e. A =0, (2) no factor mobility across countries, i.e. $b_{sj}^{hf}=0$, for $f \neq h$, (3) that the two countries have the same technology in production, i.e. $b_{sj}^{HH} = b_{sj}^{FF}$, and (4) that there exists only inter-industry trade as expressed in equation (31).

Table 2. Account Matrix of the H-O Model

			Intermediate Output				Final Goods					
			Country H		Country F		Country H		Country F		Supply	Output
			1	2	1	2	1	2	1	2	y	x
Intermediate Input	Country H	1					y_{11}^{HH}	0	y_{11}^{HF}	0	y_1^H	x_1^H
		2					0	y_{22}^{HH}	0	y_{22}^{HF}	y_2^H	x_2^H
	Country F	1					y_{11}^{FH}	0	y_{11}^{FF}	0	y_1^F	x_1^F
		2					0	y_{22}^{FH}	0	y_{22}^{FF}	y_2^F	x_2^F
							u_1^H	u_2^H	u_1^F	u_2^F		Factor
Factor Input	Country H	L	v_{11}^{HH}	v_{12}^{HH}								L^H
		K	v_{21}^{HH}	v_{22}^{HH}								K^H
	Country F	L			v_{11}^{FF}	v_{12}^{FF}						L^F
		K			v_{21}^{FF}	v_{22}^{FF}						K^F

Table 2 showed the trade flows and factor mobility for the Heckscher-Ohlin model.

There is no intermediate input in the Heckscher-Ohlin model so that equation (9) now is reduced to

$$x = y \tag{36}$$

Substituting it into equations (10) and (17), associating it with the gains-from-trade function (35), we get the set of equations of the demand-oriented-style Heckscher-Ohlin model as,

Price function: $$p = B^T w \tag{37}$$

Trade function: $$x = Cu \tag{38}$$

Resource constraint: $$v = Bx \tag{39}$$

Gains-from-trade function $$g = \begin{bmatrix} -p_1^H + \alpha p_2^H \\ p_1^F - \alpha p_2^F \end{bmatrix} \pi \tag{40}$$

Demand-Oriented
Factor Equilibrium $$v = BCu \tag{41}$$

where

$$B=\begin{bmatrix} b_{L1} & b_{L2} & 0 & 0 \\ b_{K1} & b_{K2} & 0 & 0 \\ 0 & 0 & b_{L1} & b_{L2} \\ 0 & 0 & b_{K1} & b_{K2} \end{bmatrix}, \qquad C=\begin{bmatrix} 1 & 0 & c_{11}^{HF} & 0 \\ 0 & 1-c_{22}^{FH} & 0 & 0 \\ 0 & 0 & 1-c_{11}^{HF} & 0 \\ 0 & c_{22}^{FH} & 0 & 1 \end{bmatrix}$$

$$c_{22}^{FH}=\frac{\alpha\pi}{y_2^H+\alpha\pi}, \qquad c_{11}^{HF}=\frac{\pi}{y_1^F+\pi}$$

The major improvement of the enhanced Heckscher-Ohlin model in this section is to introduce the trade function and the gain-from-trade function to this classical trade model to strength its trade side expression.

THE RICARDO MODEL BY DEMAND-ORIENTED ANALYSIS

David Ricardo's model explains international trade in terms of differences in labor productivity. It was to highlight the importance of comparative advantage. It assumes that there is only one factor, labor, in the two-country two-good system. In Ricardo's original presentation of the model, it was focused exclusively on the supply side. Only later did John Stuart Mill introduce demand into the model. We will present the Ricardo model by the framework of the demand-oriented trade model to enhance its demand analysis. We also will provide an analytical approach to prove the Ricardian law of comparative advantage in this section.

Table 3. Account Matrix of the Ricardo Model

			Intermediate Output				Final Goods					
			Country H		Country F		Country H		Country F		Supply	Output
			1	2	1	2	1	2	1	2	y	x
Intermediate Input	Country H	1					y_{11}^{HH}	0	y_{11}^{HF}	0	y_1^H	x_1^H
		2					0	y_{22}^{HH}	0	y_{22}^{HF}	y_2^H	x_2^H
	Country F	1					y_{11}^{FH}	0	y_{11}^{FF}	0	y_1^F	x_1^F
		2					0	y_{22}^{FH}	0	y_{22}^{FF}	y_2^F	x_2^F
							u_1^H	u_2^H	u_1^F	u_2^F		Factor
Factor Input	Country H	L	v_{11}^{HH}	v_{12}^{HH}								L^H
	Country F	L			v_{11}^{FF}	v_{12}^{FF}						L^F

Table 3 presents the trade flows of the Ricardo model, in which it is assumed that country H has a comparative advantage in the production of good 1 and country F has a comparative advantage in the production of good 2.

The Ricardo model can be considered as the reduced demand-oriented trade model without intermediate inputs, without the international mobility of factor, and with only inter-industry trade. It can be expressed as follows:

Price function: $$p = B^T w \qquad (42)$$

Trade function: $$x = Cu \qquad (43)$$

Resource Constraint function: $$v = Bx \qquad (44)$$

Demand-oriented Factor Equilibrium $$v = BCu \qquad (45)$$

Gains-from-trade function $$g = \begin{bmatrix} -p_1^H + \alpha p_2^H \\ p_1^F - \alpha p_2^F \end{bmatrix} \pi \qquad (46)$$

where

$$v = \begin{bmatrix} L^H \\ L^F \end{bmatrix}, \quad B = \begin{bmatrix} b_1^{HH} & b_2^{HH} & 0 & 0 \\ 0 & 0 & b_1^{FF} & b_2^{FF} \end{bmatrix}, \quad w = \begin{bmatrix} w^H \\ w^F \end{bmatrix}, \quad x = \begin{bmatrix} x_1^H \\ x_2^H \\ x_1^F \\ x_2^F \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & c_{11}^{HF} & 0 \\ 0 & 1 - c_{22}^{FH} & 0 & 0 \\ 0 & 0 & 1 - c_{11}^{HF} & 0 \\ 0 & c_{22}^{FH} & 0 & 1 \end{bmatrix}, \quad c_{22}^{FH} = \frac{\alpha\pi}{y_2^H + \alpha\pi}, \quad c_{11}^{HF} = \frac{\pi}{y_1^F + \pi}$$

We assume that the wage rate is one for both the countries to reflect the original thought of labor theory of value during the Ricardo time, in which the wage rate was evaluated by the unit equivalent labor hour. The price then is

$$p = B^T w = \begin{bmatrix} b_1^{HH} & 0 \\ b_2^{HH} & 0 \\ 0 & b_1^{FF} \\ 0 & b_2^{FF} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} b_1^{HH} \\ b_1^{HH} \\ b_1^{FF} \\ b_2^{FF} \end{bmatrix} \tag{47}$$

Substituting it into equation (46) yields:

$$g = \begin{bmatrix} -b_{L1}^{HH} + \alpha b_{L2}^{HH} \\ b_{L1}^{FF} - \alpha b_{L1}^{FF} \end{bmatrix} \pi \tag{48}$$

When trading between the two countries, the international terms of trade lies within the limits set by the price ratios in autarky.

The trade condition or criteria for the Ricardian model should be that the gains from trade for both the countries are greater than zero, i.e.,

$$g = \begin{bmatrix} -b_{L1}^{HH} + \alpha b_{L2}^{HH} \\ b_{L1}^{FF} - \alpha b_{L1}^{FF} \end{bmatrix} \pi > 0 \tag{49}$$

This implies that

$$-b_{L1}^{HH} + \alpha b_{L2}^{HH} > 0 \tag{50}$$

$$b_{L1}^{FF} - \alpha b_{L1}^{FF} > 0 \tag{51}$$

Writing equations (50) and (51) together, we obtain the following expression:

$$\frac{b_{L1}^{HH}}{b_{L2}^{HH}} < \alpha < \frac{b_{L1}^{FF}}{b_{L2}^{FF}} \tag{52}$$

Inequalities (52) are just the limits of the terms of trade in the Ricardo model. This is the essence of what the Ricardian law stated. We can say now that we did prove the Ricardian law of comparative advantage analytically by the gains-from-trade function.

Equations (42) through Equation (46) depict the general production-trade equilibriums of the Ricardo model with trade structure.

HIGHER DIMENSION MODEL

We will extend the 2 x 2 x 2 demand-oriented international trade model discussed above to the higher dimension model in this section.

Suppose that there is an integrated international economic system with *k* factors, *m* countries, and *n* goods. We will use the single superscript associated with a vector to indicate a country, the double superscripts *hf* following a matrix or its elements to indicate a transaction from country *h* to country *f*, and the double subscripts *ij* of the elements of matrix to indicate a transaction from sector *i* to sector *j*.

The *k*-factors, *m*-countries, and *n*-goods demand-oriented trade model can be presented as

Price function: $p=(I-A^T)^{-1}B^T w$ (53)

Trade function: $y = Cu$ (54)

Production Resource Constraint: $v = Bx$ (55)

Production-supply function: $X = Ax + y$ (56)

Demand-Oriented Factor Requirement $v = B(1-A)^{-1}Cu$ (57)

where

$$x=\begin{bmatrix} x^1 \\ x^2 \\ \vdots \\ x^m \end{bmatrix},\quad y=\begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^m \end{bmatrix},\quad u=\begin{bmatrix} u^1 \\ u^2 \\ \vdots \\ u^m \end{bmatrix},\quad w=\begin{bmatrix} w^1 \\ w^2 \\ \vdots \\ w^m \end{bmatrix},\quad p=\begin{bmatrix} p^1 \\ p^2 \\ \vdots \\ p^m \end{bmatrix},\quad v=\begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^m \end{bmatrix}$$

$$A=\begin{bmatrix} A^{11} & A^{12} & \cdots & A^{1m} \\ A^{21} & A^{22} & & \vdots \\ \vdots & & \ddots & \\ A^{m1} & \cdots & & A^{mm} \end{bmatrix},\quad B=\begin{bmatrix} B^{11} & B^{12} & \cdots & B^{1m} \\ B^{21} & B^{22} & & \vdots \\ \vdots & & \ddots & \\ B^{m1} & \cdots & & B^{mm} \end{bmatrix}.\quad C=\begin{bmatrix} C^{11} & C^{12} & \cdots & C^{1m} \\ C^{21} & C^{22} & & \vdots \\ \vdots & & \ddots & \\ C^{m1} & \cdots & & C^{mm} \end{bmatrix}$$

x is the [(n x m) x 1] vector of outputs for all countries of the integrated system;
p is the](n x m) x 1] vector of prices for all countries;
y is the [(n x m) x 1] vector of supply of final goods for all countries;
u is the [(n x m) x 1] vector of demand for all counties;
w^h is the [(m x k) x 1] vector of factor prices for all countries;
v^h is the [(m x k) x 1] vector of factors for all countries;
A is the [(n x m) (n x m)] technology coefficient matrix of intermediate inputs for all countries;
B is the [(m x k) (n x m)] technology coefficient matrix of the primary factors for the system;
C is the [(n x m) (n x m)] import coefficient matrix of demand of the system;
x^h is the (n x 1) vector of outputs in country h; h=1,2,…,m;
p^h is the (n x 1) vector of prices in country h;
y^h is the (n x 1) vector of supply of final goods in country h;
u^h is the (n x 1) vector of demand in country h;
w^h is the (k x 1) vector of factor prices in country h;
v^h is the (k x 1) vector of factors in country h;
A^{hf} is the (n x n) coefficient matrix of intermediate input transacted from country h to country f; its typical element a_{ij}^{hf} denotes the amount of input of good i in country h to sector j in country f per the amount of output of good j in country f; h,f are indicators of countries, h,f =1,2, ..., m; i,j are indicators of goods, i,j =1,2, …, n;
C^{hf} is the (n x n) coefficient matrix of final goods transacted from country h to country f , it is a diagonal matrix, its typical element c_{ii}^{hf} denotes the amount of good i consumed in country f made in country h per demand of the good in country f;
B^{hf} is the (k x n) is coefficient matrix of factor inputs transacted from country h to country f, its typical element b_{sj}^{hf} denotes the amount of input of factor s from country h to sector j in country f per the amount of output j in country f; s is the indicator of factor; s=1,2,…,k;
I is a identity matrix.

All dimensions in the above models are consistent. The structure of the model is flexible to hold multi-factor, multi-country, and multi-commodity. When the number of factors equals the number of commodities, it is the so-called "even number" model in the literature. In the real world, the number of commodities is much more than the number of factors; the "even number" assumption is hard to hold. The model proposed in this section provides a new approach to establish "uneven number" equilibrium of factors and commodities among multi-country economies.

The transportation costs and non-tradable goods are other realistic considerations in international trade. We may set up either transportation or non-tradable good as a sector to reflect their roles in the model. The characteristic of these two sectors or goods is that they do not take part on international trade activities directly but take part of intermediate input activities and primary factor input activities.

MEASUREMENT OF FACTOR ABUNDANCE

To measure the factor abundance of a country in the real world, we propose the following criterion equation, based on the demand-oriented trade model, as

$$r = B(I - A)^{-1}(y - u) \tag{58}$$

where r is the *[(m x k) x 1]* vector of computed factor content of net exports of the world.

The vector of supply minus the vector of demand, $(y-u)$, equals the vector of net export for all countries. The matrix $B(I-A)^{-1}$ indicates total factor requirements. Vector r is total factor content of net export.

If a country's i's computed factor content of trade is greater than zero, i.e. $r_i^h>0$, then we say that the country is abundant in that factor, because the country "exports" the factor as the result of exporting goods intensive in the factor. If a country's i's computed factor content of trade is less than zero, i.e. $r_i^h<0$, then we say that the country is scare in that factor.

Criteria r is a plain computation of factor content of net export. It allows different technologies, intermediate input, real consumption, real exports, and real imports. It may give a new clue to study the Leontief test. Equation (58) is different from Vanek (1968)'s equation, which assumed proportional consumption of each country and identical technology.

CONCLUSION

This paper introduces the demand-oriented trade model, a comprehensive international trade model, which reflects some realistic considerations in international trade practice, including intermediate input, mobile factors, intra-industry trade, taste differences, and technology differences. It demonstrates how the production of output, resource allocation, and supply of a country are impacted by the demands of other countries.

The demand-oriented trade model implies that the production and supply of a country is structurally depended on and oriented both by its domestic demand and by international demand. The equilibrium of international economies is the demand-supply equilibrium of all countries involved; the free trade will satisfy the consumption taste maximally for each of the countries; the multi-national demand-oriented development can benefit all countries.

The paper has provided an approach to process the trade-based and demand-oriented analysis in international economies. The five equilibriums of multi-national economies included in the demand-oriented trade model are general structure expressions in international trade. They do exist both in the Ricardo model and in the Heckscher-Ohlin model. The higher dimension model of the demand-oriented trade model of this paper can present the "uneven number" scenario very well.

The model presented in this paper can be a policy simulation tool to process various analyses of trade, once the data in the account matrix of international trade are collected.

The most difficult data to be collected in the model is the data of the technological coefficient matrix A; which is a world input output matrix. It may be simplified by assuming that there are no international intermediate requirements in the analyses, i.e. $A^{hf}=0$ for h not equal to f.

APPENDIX A

Transporting two sides of equation (24) yield,

$$p^T = w^T B(I-A)^{-1} \tag{A-1}$$

Substituting it into (25) yields

$$w^T B(I-A)^{-1} y = w^T v \tag{A-2}$$

Substituting equation (16) into it yields

$$v^T B(I-A)^{-1} y = v^T B(I-A)^{-1} y \tag{A-3}$$

APPENDIX B

The substitution of equation (27) into equation (28) yields:

$$B(I-A)^{-1} C(\bar{u}) l = v \tag{B-1}$$

Because

$$C(\bar{u}) l = Y l = y \tag{B-2}$$

The substitution of it into (B-1) yields,

$$B(I-A)^{-1} y = v \tag{B-3}$$

This is just equation (16).

APPENDIX C

The substitutions of equation (27) into equation (29) yield

$$w^T B(I-A)^{-1} Y = p^T Y \tag{C-1}$$

It can be reduced to

$$w^T B(I-A)^{-1} = p^T \tag{C-2}$$

The transportation of it yields

$$(I-A^T)^{-1} B^T w = p \tag{C-3}$$

It is just equation (24).

APPENDIX D

If $C = 1$ and $u = y$, equation (27) can be written as

$$Z = B(I - A)^{-1}\bar{y} \tag{D-1}$$

Because

$$V = B\bar{x} \tag{D-2}$$

and because

$$\bar{x} = (I - A)^{-1}\bar{y} \tag{D-3}$$

so that

$$V = Z \tag{D-4}$$

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