THE TEXAS LOTTERY: A PEDAGOGICAL EXAMPLE INTEGRATING CONCEPTS OF INCOME TAXATION, TIME VALUE OF MONEY, AND IRR

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ABSTRACT

This study presents a teaching exercise for a basic economics or finance class. The question posed to students is whether it is better as a lottery winner to receive a lump-sum settlement or the annuity. The exercise is designed to teach students how to integrate multiple considerations into the economically proper choice. These considerations are the implications of the progressive income tax system, the time value of money, and the implicit rate of return associated with the two alternative payment mechanisms.

INTRODUCTION

If you are purchasing a lottery ticket, is it better to receive your winnings as a lump-sum cash payment or as a series of payments over a number of years? There are two choices on a lottery that the State of Texas advertises as a $4,000,000 lottery jackpot. With option number one you receive the $4,000,000 as an annuity due spread across 25 annual payments. These payments will equal $160,000 each, for a total of $4,000,000. With option number two you receive a lump-sum cash payment of $2,000,000, 50 percent of the lottery's advertised value, which is the approximate present value of the annuity due. Remember that you are required to choose your option at the time you purchase the ticket. Also remember that taxes are involved. Now, which option is the best choice?

This lottery question is a review exercise we pose to our introductory business finance students shortly before the course's mid-term exam. This point in the class has introduced students to the time value of money, the personal income tax brackets, and internal rate of return. This "personal finance" problem requires them to integrate their knowledge of both topics to make an analytical decision in choosing one of the options. This problem will also give them additional practice in using a financial calculator or the financial tools in a computer spreadsheet. By working through the examples discussed below, students learn to include tax considerations into what initially seems to be a relatively simple time value of money problem.

Without endorsing or opposing the lottery concept itself, we recognized that the Texas lottery presents us an opportunity for a "real world" application to which many students can quickly relate. Most of our students have some familiarity with the lottery in Texas-either from the personal purchase of tickets or from being exposed to the lottery's extensive advertising campaign. For those who have purchased a ticket, they know personally that the sales clerk always asks at the time of purchase whether the purchaser wants the "cash option" or the annuity. The choice made at the time of purchase of a lottery ticket is a binding decision that determines the payment schedule to be followed if the purchaser of the ticket wins the lottery. Therefore, this is a decision that many students have made before, but perhaps without any solid financial basis for their decision. The students are immediately intrigued with this puzzle, wondering "did I do the right thing" with previous purchases.

We quickly have to put a couple of constraints on the problem, the primary one being that the students are to ignore all non-financial considerations. This is purely a wealth maximization problem. The current desire for a new sports car or a month-long vacation in Paris is to be set aside. Likewise, students are asked to ignore extreme cases such as the 97 year-old purchaser of lottery tickets who probably will not live to see 25 annual payments. The second constraint is that we must assign the pre-lottery levels of taxable income, to ensure that the students are all working on the same problem. The final constraint is that to simplify the analysis we allow the students to assume that the pre-lottery taxable income of the lottery winner will be constant for the next 25 years. This is somewhat unrealistic, but it does no harm to the pedagogy of the lesson. Factoring in a growth rate for pre-lottery income would complicate the problem without substantially increasing learning.

It should be noted that we are using the definition of "taxable income" used by the Internal Revenue Service. An individual completing a 1040 income tax form reports income and makes certain adjustments, then subtracts allowable exemptions and deductions (whether standard or itemized). The income that remains is "taxable income" and is reported on Line 39 of the Form 1040. It is "taxable income" that is taxed. In this problem we do not concern ourselves with total income, the amount of adjustments, exemptions or deductions. We start the analysis with taxable income.

Students are told to assume six specific levels of taxable income, and to determine if the family in each case is better off with the cash settlement or the annuity due. Regardless of the level of income, in this study each dollar of additional income results in an additional dollar of taxable income.

For the year 2000 our students are being told to assume six different pre-lottery taxable incomes. The first level is zero taxable income. The next four income levels are $21,925, $74,900, $133,700, and $224,900. These income levels are, respectively, the midpoints of the 15 percent, 28 percent, 31 percent and 36 percent income tax brackets. Finally $288,350 is the last income level. This is the start of the 39.6 percent bracket, so any income received from the lottery will be taxed exclusively at 39.6 percent. Table 1 presents the year 2000 personal income tax tables for a married couple filing jointly, and is the table we are using in our classes.

Table 1
2000 personal income tax table for persons married and filing jointly

Income Over	But not over	Tax of excess over	
$0	$43,850	15%	--$0
$43,850	$105,950	$6,577.50 plus 28%	--$43,850
$105,950	$161,450	$23,965.50 plus 31 %	--$105,950
$161,450	$288,350	$41,170.50 plus 36%	--$161,450
$288,350	---	$86,854.50 plus 39.6%	--$288,350

THE ANALYSIS

Regardless of which of the six levels of income the student is analyzing, the student quickly discovers that certain steps must be followed in the analysis. First the student must analyze the pre-lottery case and calculate the taxes due and the after-tax income. (Since we assign taxable income, issues of exemptions, deductions, and other confounding variables are swept aside as irrelevant to the lesson.) Then the student analyzes the implications for the change in after-tax income that results from winning the lottery, assuming they receive the annual payments. Next the student must determine the impact on taxable income if the couple wins the lottery and receives the lump-sum payment. Finally the student must find the rate of interest that equates the after-tax cash flows from the two means of payment.

As an example of the calculations, we demonstrate below the case of a married couple with a pre-lottery taxable income of $21,925. Utilizing the information from Table 1, the student determines that the couple with $21,925 in taxable income is in the 15 percent marginal tax bracket. The student then calculates taxes due to be $3,288.75 and after-tax income to $18,636.25.

Tax due = $21,925*0.15 = $3,288.75
After-tax income = $21,925-$3,288.75 = $18,636.25

With this step completed the student is ready to adjust lottery winnings to after-tax income. Since we have given the student the "pre-lottery" taxable income, the student must determine taxable income post-lottery for both an annuity due and a cash settlement.

In the second step the student is ready to address the annuity due option. Students determine that the annual annuity due payment from the lottery is $160,000 per year for 25 years. This amount is added directly to pre-lottery taxable income, then the revised values for taxes and after-tax income are determined. (This after-tax income will be important in the next step.) For the family with $21,925 in pre-lottery taxable income, the $160,000 annuity increases taxable income to $181,925. Utilizing the information from Table 1, the student finds the new marginal tax bracket to be 36 percent. The student calculates tax due to be $48,541.50, and after-tax income to be $133,383.50. The increase in after-tax income resulting from the annuity payment is $114,747.25.

Tax due = $41,170.50 + ($181,925-$161,450) *0.36 = $48,541.50
After-tax income = $181,925-$48,541.50 = $133,383.50
Increase in after-tax income = $133,383.50-$18,636.25 = $114,747.25

A major difficulty many students have is realizing that this is a marginal analysis, since they need to use marginal values later when they solve for the rate of return that will equate the annuity due and the lump-sum payment. Unless guided by the instructor, many students fail to calculate the increase in taxable income, which is crucial to analyzing the problem.

In the third step of the analysis the student adds the $2,000,000 lump-sum payment to the pre-lottery taxable income of $21,925, and increases taxable income to $2,021,925. Utilizing the information from Table 1, the student finds the new marginal tax bracket to be 39.6 percent. The student calculates tax due to be $773,350.20 and after-tax income to be $1,248,574.80. The increase in after-tax income resulting from the lump-sum lottery payment is $1,229,938.55.

Tax due = $86,854.50+(2,021,925-$288,350)*0.396 = $ 773,350.20
After-tax income = $2,021,925-$773,350.20 = $1,248,574.80
Increase in after-tax income = $1,248,574.80-$18,636.25 = $1,229,938.55

Once the student has completed the determination of the after-tax cash flows, the time value of money may finally be included in the analysis. To solve this problem the student must compare the *increase* in after-tax income that results from each option-not just simply after-tax income. The relevant values, when the student solves for the internal rate of returns in the last step, are the values of $1,229,938.55 for the lump-sum payment and $1 14,747.25 for the annuity due.

Using a financial calculator (or the financial functions in a spreadsheet as in the Appendix), the student solves for the interest rate that equates the after-tax increase in income that results from the lump-sum payment and the annuity due:

PV	= -1,229,938.55
FV	= 0
PMT	= 114,747.25

N = 25
I = ?
The internal rate of return solution (I) for this particular problem is found to be 8.9843 percent.

The final challenge for students is to interpret the meaning of the value of 8.9843 percent. For some students interpretation of the results is more difficult than the actual analysis. The solution in this particular example tells the student that 8.9843 percent is the after-tax rate of return the imaginary lottery winner must earn from the lump-sum payment to make it equal to the annuity due. Table 2 presents the internal rate of return solution for all six of the income levels used in this class exercise. The internal rates of return range from 9.4688 percent to 6.9696 percent.

Table 2
The $4,000,000 lottery

Pre-lottery taxable income	Pre-lottery marginal tax bracket	Amount of lottery payment	Method of lottery payment	Post- lottery marginal tax bracket	Internal rate of return
$0	0.00	$160,000	annuity	31.0	9.4688
$2,000,000			lump-sum	39.6	
$21,925	15.0	$160,000	annuity	36.0	8.9843
	$2,000,000		lump-sum	39.6	
$74,900	28.0	$160,000	annuity	36.0	8.2172
	$2,000,000		lump-sum	39.6	
$133,700	31.0	$160,000	annuity	39.6	7.7766
		$2,000,000	lump-sum	39.6	
$224,900	36.0	$160,000	annuity	39.6	7.2372
		$2,000,000	lump-sum		39.6
$288,350	39.6	$160,000	annuity	39.6	6.9696
		$2,000,000	lump-sum	39.6	

This value of 8.9843 is an after-tax rate of return. The before-tax rate of return must be even higher. The instructor queries the students on the likelihood of being able to earn this required rate of return. This allows the instructor to bring in a special set of data and extend the discussion. We bring in the Ibbotson (1997) financial market data that reports long-run rates of return on stocks and other assets classes. The Ibbotson data for the period 1926-1996 reports the following long-run geometric mean rates of return and standard deviations for these six asset classes:

	Rate of Return	Standard Deviation
Large company stocks	10.7%	20.3%
Small company stocks	12.6	34.1
Long-term corporate bonds	5.6	8.7
Long-term government bonds	5.1	9.2
Intermediate-term government	5.2	5.8
U.S. Treasury bills	3.7	3.3

At this point the instructor demonstrates that even with 100 percent of the return in the form of long-term capital gains which are taxed at only 10 percent, the investor would require a pre-tax return of 9.9826 percent to earn the 8.9843 percent after-tax return:

$$8.9843\% = 0.9 = 9.9826\%.$$

In light of the Ibbotson data we ask the students in the class what they believe is the probability of an investor earning 9.9826 percent on a pre-tax basis (or 8.9843 percent on an after-tax basis). Most students feel that this goal is beyond the abilities of the average investor. Putting the question differently we then ask the students to assume they put 100 percent of the lump-sum value into the stock market. Still making the over-simplifying assumption that all returns are long-term capital gains taxed at 10 percent, the expected long-run after-tax returns on stocks (based on the historical data) are

Large company stocks	10.7% * 0.9 = 9.63%
Small company stocks	12.6% * 0.9 = 1 1.34%.

Given the Ibbotson data (and despite recent stock market performance), most students recognize by this point in the class that the lump-sum payment is not nearly as attractive as they may have once thought. From the perspective of wealth maximization, most students decide that the annuity is the superior choice for this couple.

Additional observations can be made in the class once the students have performed similar analyses for the other assigned income levels, and completed Table 2. They quickly note that the annuity/lump-sum choice has different implications for persons in different marginal income brackets. Students see that the lower an individual's pre-lottery marginal tax bracket, the less attractive is the lump-sum payment. Yet the consensus opinion of our students is that as a rule of thumb, less wealthy persons are more likely to want the instant wealth of the lump-sum payment. They believe it is the upper income individuals who may really have a chance of making the lumpsum payment an attractive option.

Table 3
IRRs for various size lotteries and various pre-lottery taxable incomes

Pre-lottery taxable income	$4,000,000 lottery	$6,000,000 lottery	$8,000,000 lottery	$10,000,000 lottery
$0	9.4688	8.8993	8.5352	8.2284
$21,925	8.9843	8.5645	8.2325	7.9842
$74,900	8.2172	7.9675	7.7212	7.5724
$133,700	7.7766	7.5101	7.3759	7.2951
$224,900	7.2372	7.1483	7.1037	7.0769
$288,350	6.9696	6.9696	6.9696	6.9696

In previous semesters students have asked us two particularly intuitive "what if" questions. We have been asked about raising the assumed pre-lottery taxable income beyond the values shown. We simply asked the class to experiment with any higher level of income (beyond $288,350) of their choosing.

They quickly discovered that the internal rate of return on this problem never goes below 6.9696 percent. We have also been asked about the implications of a larger lottery. We have had the students work through that problem also. Table 3 shows the implications of four different lotteries, with the largest being valued at $10,000,000. As would be expected, the internal rate of return still has a minimum value of 6.9696 percent.

CONCLUSION

We have found the lottery problem to be an interesting exercise for students and an effective learning tool. While the subject of the exercise may seem somewhat light-hearted, we have found it to be effective in helping students with calculating after-tax cash flows, understanding concepts in the time value of money, and working with a financial calculator (or spreadsheet). Our students tell us that this exercise does help prepare them for the upcoming examination.

REFERENCES

Brigham, E. F., L. C. Gapenski & M.l C. Ehrhardt. (1999). *Financial Management: Theory and Practice,* ninth edition. Fort Worth: The Dryden Press.

Stocks, Bonds, Bills, and Inflation 1997 Yearbook (1997). Chicago: Ibbotson Associates.

APPENDIX
Excel spreadsheet for computations in exercise

	A	B	C	D	E	F	G	H
1	Income Tax Table							
2	Income At least	but less than	Tax +	Percent of Excess				
3	0	43850	0	0.15		Lotto jackpot	4000000	
4	43850	105950	6577.5	0.28		Income level	21925	
5	105950	161450	23965.5	0.31				
6	161450	288350	41170.5	0.36				
7	288350		86854.5	0.396				
8								
9Tax Consequences of Selecting 25 Payments								
10	Taxable Income	Total tax	After tax income	Lotto win- 25 years	Total income	Total tax	After tax income	Increase in after tax income
11	21925	3288.75	18636.25	160000	181925	48541.5	133383.5	114747.3
12								
13	Tax Consequences of Selecting Lump Sum Payment							
14	Income	Total tax	After tax income	Lotto win- lump sum	Total income	Total tax	After tax income	Increase in after tax income
15	21925	3288.75	18636.25	2000000	2021925	773350.2	1248575	1229939
16								
17	Income levels and corresponding IRRs							
18	Income	IRR						
19	21925	0.08984 3						

PREPARING THE SPREADSHEET

For the formulas reported below to work, the spreadsheet must be completed exactly as presented.

To prepare this spreadsheet, simply fill in rows 1-7 with the numbers as they appear in the spreadsheet. Fill in any text exactly as shown. Rows 11, 15, and 19 must have the formulas as shown below.

After this has been created the user of the spreadsheet only needs to change cells G3 and G4 to evaluate any jackpot level and any income level. The values in row 11, 15, and 19 will automatically be changed as a result of changing the values in either/both cells G3 or G4.

Row 11

A11: =G4
B11:=VLOOKUP(A11,$A$3:$D$7,3) + VLOOKUP(A11, $A$3: $D$7,4)
*(Al 1-VLOOKUP(A11, $A$3:$D$7,1))
C11: =A11-B11
D11: =$G$3/25
E11: =A11 +D11
F11: =VLOOKUP(E11,$A$3:$D$7,3) +VLOOKUP(E11,$A$3:$D$7,4)
*(Ell -VLOOKUP(E11, $A$3:$D$7,1))
G11: =-E11-F11
H11: =G11-C11

Row 15

A15: =G4
B15: =VLOOKUP(A15,$A$3:$D$7,3) +VLOOKUP(A11,$A$3:$D$7,4)
*(A15-VLOOKUP(A15, $A$3:$D$7,1))
C15: =-A15-B15
D15: =$G$3/2
E15: =A15+D15
F15: =VLOOKUP(E15,$A$3:$D$7,3) +VLOOKUP(E15,$A$3:$D$7,4)
*(E15-VLOOKUP(E15, $A$3:$D$7,1))
G15: =-E15-F15
H15: =G15-C15

Row 19

A19: =G4
B19: =RATE(25,H11,-H15,,1)